Economics 511 Problem Set
Chapter 14
Selected Answers and Hints

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1.

$$\begin{aligned} D^* &= a - b\left[\frac{a-\alpha+cy}{b+\beta}\right] + cy \\ &= \frac{a(b+\beta) - b(a-\alpha+cy) + cy(b+\beta)}{b+\beta} \\ &= \frac{a\beta + b\alpha + \beta cy}{b+\beta} \end{aligned}$$

and

$$\begin{aligned} S^* &= \alpha + \beta\left[\frac{a-\alpha+cy}{b+\beta}\right] \\ &= \frac{\alpha(b+\beta) + \beta(a-\alpha+cy)}{b+\beta} \\ &= \frac{\alpha b + \beta a + \beta cy}{b+\beta} \\ &= \frac{a\beta + b\alpha + \beta cy}{b+\beta} \end{aligned}$$

2.

$$\frac{dy}{dx} = -\frac{-e^y}{1-xe^y} = \frac{e^y}{1-xe^y}. \; (1 - xe^y \neq 0)$$

3. Competitive firm

(a) The FOC is $p - c'(y^*) = 0$ and the SOC is $-c''(y^*) < 0$. From FOC

$$\begin{aligned} \frac{\partial y^*}{\partial p} &= -\frac{1}{-c''(y)} \\ &> 0 \qquad \text{using SOC.} \end{aligned}$$

(b) The FOC is $p - c'(y^*) - t = 0$ and the SOC is $-c''(y^*) < 0$. From FOC

$$\begin{aligned} \frac{\partial y^*}{\partial t} &= -\frac{-1}{-c''(y^*)} \\ &< 0 \qquad \text{using SOC.} \end{aligned}$$

4. Let $R(q) = f(q)q$

$$\begin{aligned}
\frac{\partial \pi(q,t)}{\partial q} &= R'(q) - c'(q) - t = 0 \qquad \text{FOC} \\
\frac{\partial^2 \pi(q,t)}{\partial q^2} &= R''(q) - c''(q) < 0 \qquad \text{SOC} \\
\frac{dq^*}{dt} &= -\frac{-1}{R''(q) - c''(q)} \\
&< 0 \qquad \text{using SOC} \\
\frac{dp^*}{dt} &= \frac{dp}{dq}\frac{dq^*}{dt} \\
&= f'(q)\frac{dq^*}{dt} \\
&> 0, \qquad \text{if } f'(q) < 0.
\end{aligned}$$

5. Book publisher.

   (a) No.

   (b) Let $R(q) = f(q)q$.

$$\begin{aligned}
\max \pi &= (1-r)R(q) - c(q) \\
(1-r)R'(q) - c'(q) &= 0 \qquad \text{FOC} \\
(1-r)R''(q) - c''(q) &< 0 \qquad \text{SOC}
\end{aligned}$$

   So $R'(q) = c'(q)/(1-r) > 0$ and

$$\frac{dq^{**}}{dr} = -\frac{-R'(q)}{(1-r)R''(q) - c''(q)} < 0$$

   So, if we begin with $r = 0$ and, hence, output $= q^*$, a small increase in r will reduce output to q^{**}.

   (c) The author receives maximum royalties when revenue is maximized. This occurs when $R'(q) = 0$. If $R'(q)$ is decreasing in q then $R'(q*) > R'(\bar{q})$ implies that $q^* < \bar{q}$. With the result in (b) we have

$$q^{**} < q^* < \bar{q}.$$