

1. Linear Market Model (pages 634-636)

$$D = a - bp + cy, \quad a, b, c > 0, \quad (1)$$

$$S = \alpha + \beta p, \quad \alpha, \beta > 0; \alpha < a + cy, \quad (2)$$

where D is the quantity demanded, S is the quantity supplied, p is the price, and y is aggregate consumer income. Since $c > 0$, the good is a normal good. Endogenous: D, S, p . Exogenous: y .

(a) Equating demand and supply we get

$$a - bp^* + cy = \alpha + \beta p^* \quad (3)$$

$$a - \alpha + cy = (b + \beta)p^* \quad (4)$$

$$p^* = \frac{a - \alpha + cy}{b + \beta} \quad (5)$$

Substitute (5) into (1) and (2) and show that the resultant right hand sides become identically equal.

2. An implicit function is given by

$$y - xe^y = 0.$$

Find $\frac{dy}{dx}$.

3. A competitive firm has a cost function

$$TC = c(y)$$

and sells output y at the competitive price p .

(a) Profit maximization entails solving

$$\max_y py - c(y),$$

which yields the supply function $y^* = f(p)$. Find the comparative static derivative, $\partial y^* / \partial p$ and determine its sign.

(b) A unit tax $T = ty$ is imposed. Now the firm solves

$$\max_y py - c(y) - ty.$$

Find the comparative static derivative, $\partial y^*/\partial t$ and determine its sign.

4. A monopoly firm faces an inverse demand function given by $p = f(q)$ and has a total cost function given by $C = c(q)$. The government imposes an output tax of t per unit of output for a total tax of $T = tq$. The firm's after-tax profit is given by

$$\pi(q, t) = f(q)q - c(q) - tq.$$

Let q^* be the output that maximizes after-tax profit. Find the comparative-static derivatives

$$\frac{dq^*}{dt} \text{ and } \frac{dp^*}{dt},$$

where $p^* = f(q^*)$.

5. A book publisher is about to market a new book. The inverse demand for this new book is projected to be $p = f(q)$. The cost of producing the book is given by $C = c(q)$. Suppose that the output q^* maximizes monopoly profit $f(q)q - c(q)$.

- (a) Suppose the publisher must also pay the author a fixed royalty F . Does this change the profit-maximizing output?
- (b) Now suppose that the publisher agrees to pay the author royalties as a fraction r of book revenue. How does this change the profit-maximizing output? Find the comparative static derivatives

$$\frac{dq^{**}}{dr} \text{ and } \frac{dp^{**}}{dr}$$

where $p^{**} = f(q^{**})$.

- (c) Let \bar{q} be the output that maximizes the author's royalties. How do \bar{q} , q^* , and q^{**} compare?