Economics 511 Problem Set 14-2
Answers

D. Primont January 22, 2009

1. We can differentiate the system with respect to $\bar{E}$ to get

$$\begin{bmatrix} 1-E_Y & -E_R \\ L_Y & L_R \end{bmatrix} \begin{bmatrix} \dfrac{\partial Y^*}{\partial \bar{E}} \\ \dfrac{\partial R^*}{\partial \bar{E}} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The determinant is $\Delta = (1-E_Y)\,L_R + E_R L_Y$. The sign pattern is $(+)(-)+(-)(+)$ and so $\Delta < 0$. Then

$$\begin{bmatrix} \dfrac{\partial Y^*}{\partial \bar{E}} \\ \dfrac{\partial R^*}{\partial \bar{E}} \end{bmatrix} = \frac{1}{(1-E_Y)\,L_R + E_R L_Y} \begin{bmatrix} L_R & E_R \\ -L_Y & 1-E_Y \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\partial Y^*}{\partial \bar{E}} \\ \dfrac{\partial R^*}{\partial \bar{E}} \end{bmatrix} = \begin{bmatrix} \dfrac{L_R}{\Delta} > 0 \\ \dfrac{-L_Y}{\Delta} > 0 \end{bmatrix}$$

Alternatively, apply Cramer's Rule:

$$\frac{\partial Y^*}{\partial \bar{E}} = \frac{\begin{vmatrix} 1 & -E_R \\ 0 & L_R \end{vmatrix}}{\begin{vmatrix} 1-E_Y & -E_R \\ L_Y & L_R \end{vmatrix}} = \frac{L_R}{\Delta} > 0$$

and

$$\frac{\partial R^*}{\partial \bar{E}} = \frac{\begin{vmatrix} 1-E_Y & 1 \\ L_Y & 0 \end{vmatrix}}{\begin{vmatrix} 1-E_Y & -E_R \\ L_Y & L_R \end{vmatrix}} = \frac{-L_Y}{\Delta} > 0$$

Differentiate the system with respect to $\bar{M}$ to get

$$\begin{bmatrix} 1-E_Y & -E_R \\ L_Y & L_R \end{bmatrix} \begin{bmatrix} \dfrac{\partial Y^*}{\partial \bar{M}} \\ \dfrac{\partial R^*}{\partial \bar{M}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then

$$\begin{bmatrix} \dfrac{\partial Y^*}{\partial \bar{M}} \\ \\ \dfrac{\partial R^*}{\partial \bar{M}} \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} L_R & E_R \\ \\ -L_Y & 1-E_Y \end{bmatrix}\begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} = \begin{bmatrix} \dfrac{E_R}{\Delta} > 0 \\ \\ \dfrac{1-E_Y}{\Delta} < 0 \end{bmatrix}$$

Alternatively, using Cramer's Rule:

$$\frac{\partial Y^*}{\partial \bar{M}} = \frac{\begin{vmatrix} 0 & -E_R \\ \\ 1 & L_R \end{vmatrix}}{\begin{vmatrix} 1-E_Y & -E_R \\ \\ L_Y & L_R \end{vmatrix}} = \frac{E_R}{\Delta} > 0,$$

and

$$\frac{\partial R^*}{\partial \bar{M}} = \frac{\begin{vmatrix} 1-E_Y & 0 \\ \\ L_Y & 1 \end{vmatrix}}{\begin{vmatrix} 1-E_Y & -E_R \\ \\ L_Y & L_R \end{vmatrix}} = \frac{1-E_Y}{\Delta} < 0.$$

2. The first-order conditions were

$$\begin{aligned} L_1 &= w_1 - \lambda f_1 = 0 \\ L_2 &= w_2 - \lambda f_2 = 0 \\ L_\lambda &= y - f(x_1, x_2) = 0 \end{aligned}$$

Then, differentiating with respect to w_1 we get:

$$\begin{bmatrix} -\lambda f_{11} & -\lambda f_{12} & -f_1 \\ \\ -\lambda f_{21} & -\lambda f_{22} & -f_2 \\ \\ -f_1 & -f_2 & 0 \end{bmatrix}\begin{bmatrix} \dfrac{\partial x_1}{\partial w_1} \\ \\ \dfrac{\partial x_2}{\partial w_1} \\ \\ \dfrac{\partial \lambda}{\partial w_1} \end{bmatrix} = \begin{bmatrix} -1 \\ \\ 0 \\ \\ 0 \end{bmatrix}$$

Using Cramer's Rule

$$\frac{\partial x_1}{\partial w_1} = \frac{\begin{vmatrix} -1 & -\lambda f_{12} & -f_1 \\ 0 & -\lambda f_{22} & -f_2 \\ 0 & -f_2 & 0 \end{vmatrix}}{\left|\bar{H}\right|}$$

where

$$\left|\bar{H}\right| = \begin{vmatrix} -\lambda f_{11} & -\lambda f_{12} & -f_1 \\ -\lambda f_{21} & -\lambda f_{22} & -f_2 \\ -f_1 & -f_2 & 0 \end{vmatrix} < 0$$

Simplifying

$$\frac{\partial x_1}{\partial w_1} = \frac{f_2^2}{\left|\bar{H}\right|} < 0.$$

Next

$$\begin{aligned} \frac{\partial x_2}{\partial w_1} &= \frac{\begin{vmatrix} -\lambda f_{11} & -1 & -f_1 \\ -\lambda f_{21} & 0 & -f_2 \\ -f_1 & 0 & 0 \end{vmatrix}}{\left|\bar{H}\right|} \\ &= \frac{-f_1 f_2}{\left|\bar{H}\right|} > 0 \end{aligned}$$

Differentiating with respect to w_2 we get:

$$\begin{bmatrix} -\lambda f_{11} & -\lambda f_{12} & -f_1 \\ -\lambda f_{21} & -\lambda f_{22} & -f_2 \\ -f_1 & -f_2 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial x_1}{\partial w_2} \\ \frac{\partial x_2}{\partial w_2} \\ \frac{\partial \lambda}{\partial w_2} \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Using Cramer's Rule

$$\frac{\partial x_1}{\partial w_2} = \frac{\begin{vmatrix} 0 & -\lambda f_{12} & -f_1 \\ -1 & -\lambda f_{22} & -f_2 \\ 0 & -f_2 & 0 \end{vmatrix}}{\left|\bar{H}\right|} = \frac{-f_1 f_2}{\left|\bar{H}\right|} > 0$$

Finally,

$$\frac{\partial x_2}{\partial w_2} = \frac{\begin{vmatrix} -\lambda f_{11} & 0 & -f_1 \\ \\ -\lambda f_{21} & -1 & -f_2 \\ \\ -f_1 & 0 & 0 \end{vmatrix}}{\left|\bar{H}\right|} = \frac{f_1^2}{\left|\bar{H}\right|} < 0.$$