Economics 511 Problem Set 14-3

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1. A competitive firm maximizes profit by solving

$$\pi(p, \mathbf{w}) = \max_{y,\mathbf{x}} py - \mathbf{w} \cdot \mathbf{x} \text{ subject to } y = f(\mathbf{x}),$$

where the optimal value function π is called the profit function, p is the price of output, y is output, $\mathbf{w}$ is a vector of input prices, $\mathbf{x}$ is a vector of inputs, and f is the production function. Use the envelope theorem to find

$$\frac{\partial \pi(p, \mathbf{w})}{\partial p} \text{ and } \frac{\partial \pi(p, \mathbf{w})}{\partial w_i}, i = 1, ..., n.$$

2. A competitive firm minimizes cost by solving

$$c(\mathbf{w}, y) = \min_{x} \mathbf{w} \cdot \mathbf{x} \text{ subject to } y = f(\mathbf{x}),$$

where the optimal value function c is called the cost function and all other notation is defined in the previous problem. Use the envelope theorem to find

$$\frac{\partial c(\mathbf{w}, y)}{\partial y}.$$

3. The standard income-leisure model is given by the maximization problem

$$v(w, p, V, T) = \max_{\ell, c} u(\ell, c) \text{ subject to } w\ell + pc = wT + V$$

where u is the worker's utility function over leisure hours (ℓ) and real consumption (c). The wage rate is w and the price of consumption is p. T is the total hours available that are allocated between leisure hours and hours worked ($h = T - \ell$). V is nonlabor income. The optimal value function v is called an indirect utility function. The Lagrangian for this problem is

$$L = u(\ell, c) + \lambda \left(wT + V - w\ell - pc \right).$$

Find

$$\frac{\partial v(w, p, V, T)}{\partial w} \text{ and } \frac{\partial v(w, p, V, T)}{\partial V}.$$

Use the two results to eliminate λ^*.