Economics 511 Problem Set 18 Answers

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1. Solve each of the following difference equations. Also, solve for the steady state if it exists and indicate whether or not y_t converges to the steady state.

   (a) $y_{t+1} = 0.5y_t + 10, \quad y_0 = 24.$
   The steady state solution is the solution to

   $$\bar{y} = 0.5\bar{y} + 10; \quad \bar{y} = 20$$

   The solution is

   $$\begin{aligned} y_t &= (0.5)^t (24 - 20) + 20 \\ &= (0.5)^t (4) + 20 \end{aligned}$$

   This converges.

t	0	1	2	3	4	5
y_t	24	22	21	20.5	20.25	...

   (b) $y_{t+1} = y_t + 2, \quad y_0 = 4$
   The solution is $y_t = 2t + 4$. There is no steady state value.

t	0	1	2	3	4	5
y_t	4	6	8	10	12	...

   (c) $y_t = 4y_{t-1} - 12, \quad y_0 = 5$
   The steady state solution solves $\bar{y} = 4\bar{y} - 12$. Thus $\bar{y} = 4$. The solution is

   $$y_t = (4)^t + 4$$

   It does not converge.

t	0	1	2	3	4	5
y_t	5	8	20	68	260	...

2. In the labor market for new engineering graduates, demand depends on the current wage. Demand is given by:

   $$w_t = \alpha - \beta E_t, \quad \alpha > 0, \beta > 0$$

   where w_t is the wage at time t and E_t is the number of new engineers at time t. The supply depends on the wage in the previous period (since the decision to major in engineering occurs in the time period before new engineers enter the labor market.) Supply is given by

   $$w_{t-1} = \gamma + \delta E_t, \quad \gamma > 0, \delta > 0.$$

(a) Show that

$$w_{t+1} = \frac{-\beta}{\delta} w_t + \frac{\alpha\delta + \beta\gamma}{\delta}$$

describes the time path of the wage in this market.

Solve each equation for E_t.

$$\begin{aligned} E_t &= \frac{\alpha}{\beta} - \frac{1}{\beta} w_t \\ E_t &= \frac{1}{\delta} w_{t-1} - \frac{\gamma}{\delta} \end{aligned}$$

Thus

$$\begin{aligned} \frac{\alpha}{\beta} - \frac{1}{\beta} w_t &= \frac{1}{\delta} w_{t-1} - \frac{\gamma}{\delta} \\ \alpha - w_t &= \frac{\beta}{\delta} w_{t-1} - \frac{\beta\gamma}{\delta} \\ w_t &= \frac{-\beta}{\delta} w_{t-1} + \alpha + \frac{\beta\gamma}{\delta} \\ w_t &= \frac{-\beta}{\delta} w_{t-1} + \frac{\alpha\delta + \beta\gamma}{\delta} \end{aligned}$$

Advancing time by one period

$$w_{t+1} = \frac{-\beta}{\delta} w_t + \frac{\alpha\delta + \beta\gamma}{\delta}$$

(b) Find the steady-state wage.

$$\begin{aligned} \bar{w} &= \frac{-\beta}{\delta} \bar{w} + \frac{\alpha\delta + \beta\gamma}{\delta} \\ \delta\bar{w} &= -\beta\bar{w} + \alpha\delta + \beta\gamma \\ \bar{w} &= \frac{\alpha\delta + \beta\gamma}{\delta + \beta} \end{aligned}$$

(c) What restriction on the parameters guarantees that the wage will converge to its steady-state value?

The appropriate restriction is that $-1 < -\beta/\delta < 0$. This means that $0 < \beta/\delta < 1 \Rightarrow \beta < \delta$. The demand curve for labor is flatter than the supply curve.

(d) What is the nature of the convergent time path? Is it oscillating, monotonic, chaotic, or indeterminate?

Since $-1 < -\beta/\delta < 0$, the time path is oscillating. The wage rate alternately over- and under-shoots the steady-state value.