Economics 511 Problem Set 19
Answers

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1. (Ex. 1, page 798) See text.

2. (Ex. 4, page 798) Given:

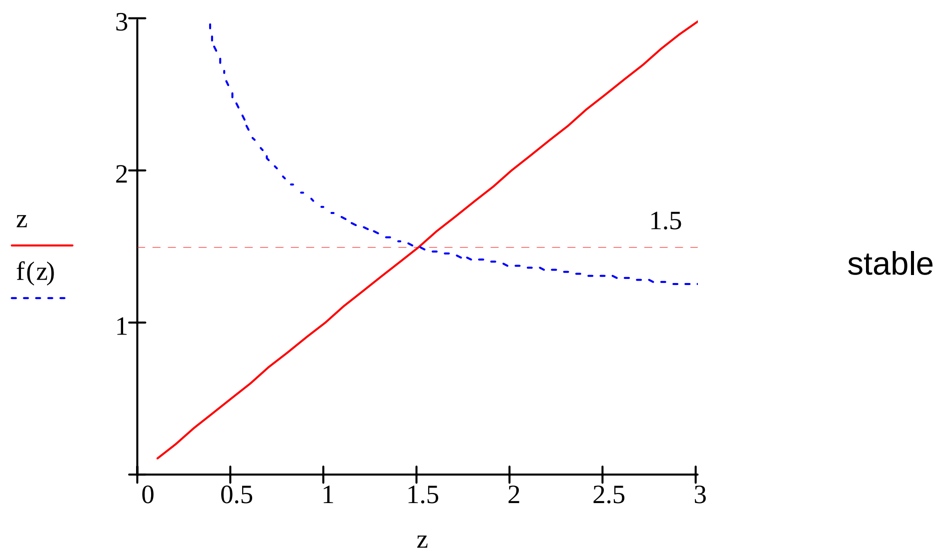
$$y_{t+1} = 1 + \frac{3}{4y_t}.$$

Solve

$$\begin{aligned} y &= 1 + \frac{3}{4y} \\ 4y^2 &= 4y + 3 \\ 4y^2 - 4y - 3 &= 0 \\ y &= \frac{4 \pm \sqrt{16 - 4(4)(-3)}}{8} = \frac{4 \pm 8}{8} \\ &= \left(-\frac{1}{2}, \frac{3}{2}\right) \end{aligned}$$

$$\frac{dy_{t+1}}{dy_t} = \frac{-3}{4y_t^2} = \frac{-3}{4\left(\frac{9}{4}\right)} = -\frac{1}{3}.$$

The equation is locally stable at 3/2.



3. Given:

$$y_{t+1} = \frac{1}{2}\left(y_t + \frac{c}{y_t}\right), \qquad c > 0$$

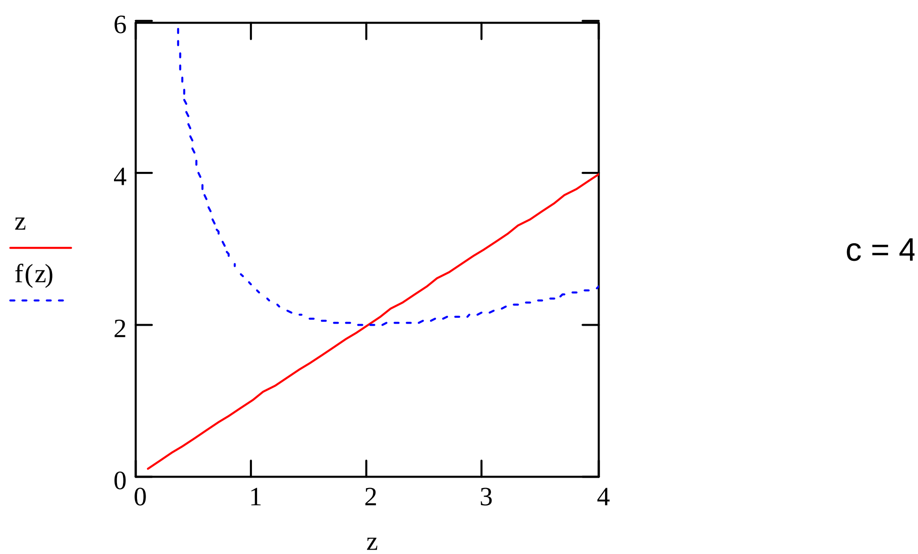
Solve

$$\begin{aligned} y &= \frac{1}{2}\left(y + \frac{c}{y}\right) \\ 2y &= y + \frac{c}{y} \\ y &= \frac{c}{y} \\ y &= \pm\sqrt{c}. \end{aligned}$$

Taking the positive square root,

$$\frac{dy_{t+1}}{dy_t} = \frac{1}{2}\left(1 - \frac{c}{y_t^2}\right) = \frac{1}{2}\left(1 - \frac{c}{y_t^2}\right) = 0 \text{ at } y_t^2 = c.$$

Locally stable.



This converges fairly quickly.

4. (Ex. 6, page 808) Given:

$$C_{t+1} = a + bY_t^{\alpha}, 0 < \alpha < 1; 0 < b < 1$$

and

$$Y_t = C_t + I$$

we get

$$C_{t+1} = a + b\left(C_t + I\right)^{\alpha}$$

Solve

$$\bar{C} = a + b(\bar{C}+I)^{\alpha} \Rightarrow \bar{C} - a = b(\bar{C}+I)^{\alpha} = b(\bar{C}+I)^{\alpha-1}\left(\bar{C}+I\right)$$

$$\frac{\bar{C}-a}{\bar{C}+I} = b(\bar{C}+I)^{\alpha-1}$$

$$\frac{dC_{t+1}}{dC_t} = \alpha b(C_t+I)^{\alpha-1}\Big|_{C_t=\bar{C}} = \alpha\frac{\bar{C}-a}{\bar{C}+I} < 1.$$

Locally stable at $\bar{C}$.

$a := 10 \quad b := 1 \quad I := 10 \quad \alpha := .9 \quad f(z) := a + b\cdot(z+I)^{\alpha}$