Economics 511 Problem Set 22
Nonlinear First-Order Differential Equations

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1. (Exercise 1, page 887) Use a phase diagram and theorem 22.2 to conduct a qualitative analysis of $\dot{y} = -y + y^2 + 3/16$.

2. (Exercise 5, page 887) Quantity demanded in a market is given by

$$q^d = p^{-2}$$

and quantity supplied is given by

$$q^s = 8p.$$

If price adjusts according to $\dot{p} = \alpha(q^d - q^s)$, where $\alpha > 0$ is a constant, conduct a qualitative analysis of the dynamics of market price.

3. Quantity demanded in a market is given by

$$q^d = D(p); \qquad D'(p) < 0$$

and quantity supplied is given by

$$q^s = S(p); \qquad S'(p) \geq 0.$$

If price adjusts according to $\dot{p} = \alpha(D(p) - S(p))$, where $\alpha > 0$ is a constant, conduct a qualitative analysis of the dynamics of market price.

4. Let y be the proportion of a population that has already been infected with a highly contagious flu as of time t. Let z (a constant) be the fraction of the population that is either inoculated against the flu or is otherwise immune to the flu. Thus $0 \leq y \leq 1 - z$. The rate of change of y is proportional to 1) y (the more people with the flu the faster it spreads) and 2) $1 - y - z$, the proportion that are vulnerable to contracting the flu. The differential equation is

$$\dot{y} = ay(1 - y - z) = a(1 - z)y - ay^2.$$

Conduct a qualitative analysis of the dynamics of flu infection.

5. (Exercise 6, page 887) Let y be the *stock* of carbon dioxide in the atmosphere. Let $x > 0$ (a constant) be the *flow* of carbon dioxide emissions that arise from industrial activity. Assume that the dynamics of y are given by

$$\dot{y} = x - y^a, \qquad x > 0.$$

where the term y^a measures the earth's capacity to remove carbon dioxide from the atmosphere (and absorb it into plant life and oceans). Conduct a qualitative analysis of this model, first for the case that $a > 0$ and then for the case $a < 0$. Comment on your results.

6. (Exercise 1, page 893) Solve
$$\dot{y} + 2y = \frac{3}{y}.$$

7. Find the explicit solution to the flu infection problem.

8. Solve
$$\dot{y} = \frac{t^2}{y(1+t^3)}$$

9. The slope of a consumer's indifference curve is given by
$$\left.\frac{dx_2}{dx_1}\right|_{u\,=\,\text{constant}} = -\frac{\partial u(x_1,x_2)/\partial x_1}{\partial u(x_1,x_2)/\partial x_2}.$$
Suppose
$$\left.\frac{dx_2}{dx_1}\right|_{u\,=\,\text{constant}} = -\frac{\alpha x_1^{\alpha-1} x_2^{\beta}}{\beta x_1^{\alpha} x_2^{\beta-1}}.$$
This is a nonlinear, first-order differential equation. Solve to find the utility function. (Hint: First simplify the RHS. The result will be a differential equation that is separable.)