Economics 511 Problem Set 24
Simultaneous Systems of Differential Equations

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1. Solve each of the following systems of linear differential equations using (*i*) the substitution method and (*ii*) the direct method. (See 24.1, Exercise 3. (i) and (iii), page 951.)

   (a)
   $$\dot{y}_1 = -2y_1 + 2y_2 + 12 \qquad y_1(0) = -2$$
   $$\dot{y}_2 = y_1 - 3y_2 - 12 \qquad y_2(0) = 5$$

   (b)
   $$\dot{y}_1 = 2y_1 - 2y_2 + 5 \qquad y_1(0) = 2.5$$
   $$\dot{y}_2 = 2y_1 + 2y_2 + 1 \qquad y_2(0) = -1$$

2. Solve the following linear differential equation system, draw the phase diagram, and find the equation for the saddle path. If $y_1(0) = 8$, what value must be chosen for $y_2(0)$ to ensure that the system converges to the steady state? (See 24.2, Exercise 3, page 975.)

   $$\begin{aligned} \dot{y}_1 &= 2y_1 - 9y_2 + 35 \\ \dot{y}_2 &= -3y_1 - 4y_2 + 70 \end{aligned}$$

3. In the following nonlinear differential equation system, $I(t)$ is a firm's investment at time t, $K(t)$ is its capital stock at time t, δ is the rate of depreciation of capital, and α is a parameter of the firm's production function with $0 < \alpha < 1$. (See Review Exercises, Exercise 5, page 996)

   $$\begin{aligned} \dot{I} &= \delta I - \frac{\alpha K^{\alpha-1}}{2} \\ \dot{K} &= I - \delta K \end{aligned}$$

   Find the steady-state point, show that it is a saddle point, and construct the phase diagram.