Economics 511 Problem Set 14-2

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1. In the IS-LM model, aggregate endogenous expenditure, E, is a function of aggregate income, Y, and the interest rate, R. Exogenous expenditure is $\bar{E}$. Equilibrium values of Y and R must satisfy the condition that aggregate income equals aggregate expenditure:

$$Y = \bar{E} + E(Y, R), \quad 0 < E_Y < 1, \quad E_R < 0.$$

In the money market, the demand for money is a function, $L(Y, R)$, of aggregate income and the interest rate. The exogenous money supply is $\bar{M}$. Money market equilibrium entails

$$L(Y, R) = \bar{M}, \quad L_Y > 0, \quad L_R < 0.$$

The combined system is

$$\begin{aligned} Y^* - \bar{E} - E(Y^*, R^*) &= 0 \\ L(Y^*, R^*) - \bar{M} &= 0 \end{aligned}$$

Find the comparative static derivatives

$$\frac{\partial Y^*}{\partial \bar{E}}, \quad \frac{\partial R^*}{\partial \bar{E}}, \quad \frac{\partial Y^*}{\partial \bar{M}}, \quad \frac{\partial R^*}{\partial \bar{M}},$$

and determine their signs.

2. Cost Minimization: The cost minimization problem for a firm that employs two inputs to produce one output looks like this:

$$\min_{x_1,x_2} w_1 x_1 + w_2 x_2 \text{ subject to } f(x_1,x_2) = y$$

The Lagrangian is

$$L = w_1 x_1 + w_2 x_2 + \lambda \left(y - f(x_1,x_2) \right)$$

and the first order conditions are

$$\begin{aligned} L_1 &= w_1 - \lambda f_1 = 0 \\ L_2 &= w_2 - \lambda f_2 = 0 \\ L_\lambda &= y - f(x_1,x_2) = 0 \end{aligned}$$

The second order condition is

$$\begin{vmatrix} L_{11} & L_{12} & L_{1\lambda} \\ L_{21} & L_{22} & L_{2\lambda} \\ L_{\lambda 1} & L_{\lambda 2} & L_{\lambda\lambda} \end{vmatrix} = \begin{vmatrix} -\lambda f_{11} & -\lambda f_{12} & -f_1 \\ -\lambda f_{21} & -\lambda f_{22} & -f_2 \\ -f_1 & -f_2 & 0 \end{vmatrix} < 0.$$

The solution is $(x_1^*, x_2^*) = (x_1(w_1,w_2,y), x_2(w_1,w_2,y))$. Find the matrix of comparative static derivatives:

$$\begin{bmatrix} \dfrac{\partial x_1^*}{\partial w_1} & \dfrac{\partial x_1^*}{\partial w_2} & \dfrac{\partial x_1^*}{\partial y} \\ \\ \dfrac{\partial x_2^*}{\partial w_1} & \dfrac{\partial x_2^*}{\partial w_2} & \dfrac{\partial x_2^*}{\partial y} \end{bmatrix}.$$