Economics 511 Problem Set 15

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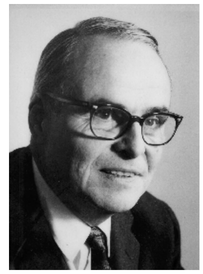

Harold Kuhn   Albert Tucker

1. Suppose $f(x)$ is a concave function. Show that $g(x) = -f(x)$ is a convex function.

2. Suppose $g(x)$ is a convex function. Show that $f(x) = -g(x)$ is a concave function.

3. Let $h(x)$ be a linear combination of a collection of functions, $g^k(x), k = 1, ..., K$, i.e.,
$$h(x) = \sum_{k=1}^{K} a_k g^k(x).$$
Assuming that $a_k \geq 0, k = 1, ..., K$, show that
   (a) $h(x)$ is a concave function if each $g^k, k = 1, ..., K$, is a concave function.
   (b) $h(x)$ is a convex function if each $g^k, k = 1, ..., K$, is a convex function.

4. For each of the following problems find the solutions that satisfy the appropriate Kuhn-Tucker (KT) conditions. The solution includes values of the multipliers as well as values of the choice variables.
   (a)
$$\begin{array}{rl} \text{maximize} & \ln x_1 + \ln x_2 \\ \text{subject to} & x_1 + x_2 \leq 25, \quad x_1 \leq 10 \\ & x_1, x_2 \geq 0 \end{array}$$
   (b)
$$\begin{array}{rl} \text{minimize} & x_1 + x_2 \\ \text{subject to} & x_1^{1/2} x_2^{1/2} \geq 100, \quad x_1 \geq 5 \\ & x_1, x_2 \geq 0 \end{array}$$

5. Consider the problem:

$$\max x_1 + \ln x_2 \text{ subject to } m - px_1 - x_2 \geq 0, x_1 \geq 0, x_2 \geq 0,$$

where $p > 0$ and $m > 0$. Verify that the objective function is a concave function. Find and *characterize* all possible solutions to this problem.