

## Problem Set 1: Technology and Profit

1. (Varian, Problem 1.10) The technology  $Y$  is *additive* if

$$\mathbf{y} \in Y \text{ and } \mathbf{y}' \in Y \Rightarrow \mathbf{y} + \mathbf{y}' \in Y.$$

The technology is *divisible* if

$$\mathbf{y} \in Y \text{ and } 0 \leq t \leq 1 \Rightarrow t\mathbf{y} \in Y.$$

Show that if a technology is both additive and divisible, then  $Y$  must be convex and exhibit constant returns to scale.

2. Consider the production function

$$y = Ax_1^a x_2^b, \quad a > 0, b > 0, a + b < 1.$$

- (a) Calculate the elasticity of scale.
- (b) Add a third input,  $z$ , in such a way that the elasticity of scale is 1. How is this procedure justified?
3. A simple candidate for a production function with one input is:  $y = \ln x$ .
- (a) Why is this a bad choice?
- (b) Consider a better candidate given by:  $y = \ln(x + 1)$ . Why is this a better choice?
- (c) Convert  $y$  and  $x$  into netputs  $y_2$  and  $y_1$ , respectively, and specify the netput set  $Y$  in such a way that  $Y$  satisfies netput monotonicity.
4. Find the profit function for the technology given by  $Y = \{(y_1, y_2) : y_1 + y_2^2 \leq 0\}$ .
5. State the *Weak Axiom of Profit Maximization*. Now consider the following data for a competitive firm. There are two observations of the firm's vectors of netputs and their corresponding prices.

Period	$p_1$	$p_2$	$y_1$	$y_2$
1	2	8	-4	2
2	6	12	-1	1

Is this data consistent with profit-maximizing behavior?

6. Show that if the technology exhibits nondecreasing returns to scale then  $\pi(\mathbf{p}) = 0$  or  $\pi(\mathbf{p}) = +\infty$ .

7. Suppose the Weak Axiom of Profit Maximization (WAPM) is not satisfied for a set of data given by  $(\mathbf{p}^t, \mathbf{y}^t), t = 1, \dots, T$ . Why might WAPM fail? List as many reasons as possible and briefly justify each reason.
8. Given a technology set  $Y$ , the set of all feasible netput vectors, that is not directly observable, one can attempt to find inner and outer approximations to  $Y$  using an appropriate set of data.
  - (a) What is meant by the term “inner approximation”? What assumptions must be made to ensure that such an approximation can be made? What data are needed?
  - (b) What is meant by the term “outer approximation”? What assumptions must be made to ensure that such an approximation can be made? What data are needed?
9. Consider the production function:  $y = x + x^{1/2}$ . Find the profit function.
10. (Varian, 3.2, p. 48) Consider the technology described by  $y = 0$  for  $x \leq 1$  and  $y = \ln x$  for  $x > 1$ . Calculate the profit function for this technology.
11. A firm is a competitive profit maximizer that produces widgets. What happens to the quantity of widgets supplied when
  - (a) all prices in the economy double?
  - (b) the price of widgets rises.
  - (c) the price of non-skilled labor rises?