Economics 540B Spring 2009

Homework 2 Cost Short Answers

1. $y = 4$

2. Since $(\mathbf{x}^*, \mathbf{y}^*)$ is profit maximizing

$$(\mathbf{x}^*, \mathbf{y}^*) \in T \text{ and } \mathbf{py}^* - \mathbf{wx}^* \geq \mathbf{py} - \mathbf{wx} \text{ for all } (\mathbf{x}, \mathbf{y}) \text{ in } T.$$

   This implies that

$$(\mathbf{x}^*, \mathbf{y}^*) \in T \text{ and } \mathbf{py}^* - \mathbf{wx}^* \geq \mathbf{py}^* - \mathbf{wx} \text{ for all } \mathbf{x} \text{ such that } (\mathbf{x}, \mathbf{y}^*) \text{ is in } T.$$

   This implies that

$$(\mathbf{x}^*, \mathbf{y}^*) \in T \text{ and } \mathbf{wx}^* \leq \mathbf{wx} \text{ for all } \mathbf{x} \text{ such that } (\mathbf{x}, \mathbf{y}^*) \text{ is in } T,$$

   and so $\mathbf{x}^*$ is the cost-minimizing choice when producing $\mathbf{y}^*$ at input prices, $\mathbf{w}$.

3. This was covered in class. By using the Kuhn-Tucker theorem we get

$$c(y) = \begin{cases} \dfrac{y^2}{2} & 0 \leq y \leq 1 \\ y - \dfrac{1}{2} & y > 1 \end{cases}$$

4. Homogeneity of degree zero implies that $a = 1/2$ and $c = -1/2$. Symmetry $(\partial x_1/\partial w_2 = \partial x_2/\partial w_1)$ implies that $b = 3$.

5. Check the Weak Axiom of Cost Minimization (WACM). We get

$$100 = w^A x^A \leq w^A x^B = 105$$

   and

$$115 = w^B x^B \nleq w^B x^A = 100$$

   so WACM is violated.

6.

   (a) Since $y^2 < y^1$, WACM implies that

$$\begin{aligned} w^2 x^2 &\leq w^2 x^1 \Rightarrow 2\,(40) + 3\,(30) \leq 2\,(20) + 3\,(60) \\ &\Rightarrow 170 \leq 220. \end{aligned}$$

   WACM is satisfied.

(b) WAPM implies that

$$p^1y^1 - w^1x^1 \geq p^1y^2 - w^1x^2 \tag{1}$$

and

$$p^2y^2 - w^2x^2 \geq p^2y^1 - w^2x^1 \tag{2}$$

Inequality (1) is satisfied since

$$6\,(100) - 1\,(20) - 4\,(60) = 340 \geq 320 = 6\,(80) - 1\,(40) - 4\,(30)\,.$$

However, inequality (2) is not satisfied since

$$5\,(80) - 2\,(40) - 3\,(30) = 230 \ngeq 280 = 5\,(100) - 2\,(20) - 3\,(60)$$

So WAPM is violated.

7. This was done in class.

8. A function f is strictly concave on its convex domain if

$$f(kx^1 + (1-k)x^2) > kf(x^1) + (1-k)f(x^2)$$

for all k, $0 < k < 1$, and for all x^1 and for all x^2 in the convex domain of f.

Choose any two price vectors, w^1 and w^2, in the convex domain of $c(\cdot, y)$ such that $w^2 = \theta w^1$.

$$\begin{aligned}
c(kw^1 + (1-k)w^2, y) &= c(kw^1 + (1-k)\theta w^1, y) \\
&= c([k + (1-k)\theta]\, w^1, y) \\
&= [k + (1-k)\theta]\, c(w^1, y) \quad \text{by homogeneity} \\
&= kc(w^1, y) + (1-k)\theta c(w^1, y) \quad \text{expanding} \\
&= kc(w^1, y) + (1-k)c(\theta w^1, y) \quad \text{by homogeneity} \\
&= kc(w^1, y) + (1-k)c(w^2, y)
\end{aligned}$$

Thus, the condition for strict concavity fails along rays from the origin in price space.