

Problem Set 2 Cost

- (Varian, 4.8, page 63) A firm has a production function given by $y = x_1x_2$. If the minimum cost of production at $w_1 = w_2 = 1$ is equal to 4, what is y equal to?
- A firm maximizes profit over the technology set

$$T = \{(\mathbf{x}, \mathbf{y}) : \mathbf{x} \in R_+^N, \mathbf{y} \in R_+^M, \mathbf{x} \text{ can produce } \mathbf{y}\},$$

by choosing $(\mathbf{x}^*, \mathbf{y}^*)$ when the corresponding price vectors are (\mathbf{w}, \mathbf{p}) . Show that \mathbf{x}^* is the cost-minimizing choice when producing \mathbf{y}^* at input prices, \mathbf{w} .

- [Varian, Problem 4.3, page 63] A firm has two plants with cost functions $c_1(y_1) = y_1^2/2$ and $c_2(y_2) = y_2$. Total output is $y = y_1 + y_2$. Find the cost function, $c(y)$, for the firm.
- [Varian, Problem 5.6, page 78] Consider a firm with conditional factor demand functions of the form

$$\begin{aligned} x_1 &= 1 + 3w_1^{-1/2}w_2^a \\ x_2 &= 1 + bw_1^{1/2}w_2^c. \end{aligned}$$

Output has been set equal to 1 for convenience. What are the values of the parameters a, b , and c and why?

- The following table shows two observations on factor demand, x_1, x_2 , factor prices, w_1, w_2 , and scalar output, y , for a firm. Is the data in this table consistent with cost-minimizing behavior?

Obs	y	w_1	w_2	x_1	x_2
A	10	3	2	20	20
B	10	4	1	25	15

Be sure to clearly justify your answer!

- You are given the following data for a competitive firm.

Obs	p	w_1	w_2	y	x_1	x_2
1	6	1	4	100	20	60
2	5	2	3	80	40	30

- Is this data consistent with the hypothesis of cost minimization? Explain.

- (b) Is this data consistent with the hypothesis of profit maximization? Explain.
7. A competitive firm's cost function is concave in input prices. Therefore, the Hessian matrix of the cost function is negative semi-definite. Prove that this Hessian matrix is singular and that, therefore, the Hessian matrix cannot be negative definite.
 8. Prove directly, without taking derivatives, that the cost function cannot be strictly concave.