Economics 540B Spring 2009

Problem Set 3 Duality

1. The cost function is defined by

$$c(\mathbf{w}, y) = \min_{\mathbf{x}} \{\mathbf{wx} : \mathbf{x} \in V(y)\}$$

where $V(y)$ is the input requirement set.

   (a) When is it possible to exactly construct $V(y)$ given the cost function, $c(\mathbf{w}, y)$? How is it done?

   (b) Regardless of whether one can exactly construct $V(y)$ from the cost function, show that we can always construct a set, $V^*(y)$, such that $V(y) \subseteq V^*(y)$.

   (c) Here is a quote from your textbook. "*the cost function of a firm summarizes all of the economically relevant aspects of its technology.*" Varian (1992, page 84) Explain what Varian means by this statement. Part of your answer should include an explanation of the term "*economically relevant aspects of its technology*". Be sure to justify the entire quoted statement.

2. A firm's cost function is given by

$$c(\mathbf{w}, y) = \gamma_1 w_1 + 2w_1^{1/2} w_2^{1/2} y + \gamma_2 w_2, \qquad \gamma_1 > 0, \gamma_2 > 0.$$

   (a) Show that this cost function is monotonic and concave in input prices.

   (b) Find the firm's production function.

3. Find the input requirement set, $V(y)$, for each of the following cost functions. You should show all of your work and justify each step in your reasoning.

   (a) $c(\mathbf{w}, y) = \min\left\{\frac{w_1}{a_1}, \frac{w_2}{a_2}\right\} y$

   (b) $c(\mathbf{w}, y) = (b_1 w_1 + b_2 w_2)\, y$

   (c) $c(\mathbf{w}, y) = \gamma_1 w_1 + 2w_1^{1/2} w_2^{1/2} y + \gamma_2 w_2$

4. According the article by Primont and Sawyer (1993),

$$c(\mathbf{w}, y) = \min_{\mathbf{x}} \{\mathbf{wx} : f(\mathbf{x}) \geq y\} \tag{1}$$

if and only if

$$f(\mathbf{x}) = \min_{\mathbf{w}, y} \{y : \mathbf{wx} \leq c(\mathbf{w}, y)\}. \tag{2}$$

The corresponding Lagrangian functions are

$$L = \mathbf{wx} + \lambda\left(y - f(\mathbf{x})\right) \tag{3}$$

and

$$L^{*} = y + \kappa\left(\mathbf{wx} - c(\mathbf{w}, y)\right).$$

(a) Apply the envelope theorem to find

$$\frac{\partial c(\mathbf{w}, y)}{\partial w_i}, i = 1, ..., n \text{ and } \frac{\partial c(\mathbf{w}, y)}{\partial y}$$

from (1) and to find

$$\frac{\partial f(x)}{\partial x_i}, i = 1, ..., n,$$

from (2).

(b) Give *economic* interpretations for the results in (a).