Economics 540B Spring 2009

Problem Set 4 Efficiency Short Answers

1. Distance Functions

(a)

$$D_0(x,y) = \frac{y}{\min\left\{\frac{x_1}{a}, \frac{x_2}{a_2}\right\}}, \qquad D_i(y,x) = \frac{\min\left\{\frac{x_1}{a}, \frac{x_2}{a_2}\right\}}{y}$$

(b)

$$D_0(x,y) = \frac{y}{x_1^{\alpha_1} x_2^{\alpha_2}}, \qquad D_i(y,x) = \frac{x_1^{\alpha_1/\alpha} x_2^{\alpha_2/\alpha}}{y^{1/\alpha}}, \quad \alpha = \alpha_1 + \alpha_2.$$

(c)

$$D_0(x,y) = \frac{\max\{y_1/b_1, y_2/b_2\}}{\min\{x_1/a_1, x_2/a_2\}}, \qquad D_i(y,x) = \frac{\min\{x_1/a_1, x_2/a_2\}}{\max\{y_1/b_1, y_2/b_2\}}$$

(d)

$$D_0(x,y) = \frac{(y_1^\rho + y_2^\rho)^{1/\rho}}{x_1^{\alpha_1/\rho} x_2^{\alpha_2/\rho}}, \qquad D_i(y,x) = \frac{x_1^{\alpha_1/\alpha} x_2^{\alpha_2/\alpha}}{(y_1^\rho + y_2^\rho)^{1/\alpha}} \quad \alpha = \alpha_1 + \alpha_2.$$

(e)

$$D_0(x,y) = \frac{y}{\min\{a_1 x_1, b_1 x_1 + b_2 x_2\}}, \quad D_i(y,x) = \frac{\min\{a_1 x_1, b_1 x_1 + b_2 x_2\}}{y}.$$

(f)

$$\begin{aligned} D_0(x,y) &= \inf_\theta \left\{\theta > 0 : \frac{y_2}{\theta} \leq \frac{y_1}{\theta} \leq x\right\} \\ &= \inf_\theta \left\{\theta > 0 : \frac{y_1}{x} \leq \theta\right\} \text{ if } y_2 \leq y_1 \\ &= \frac{y_1}{x} \text{ if } y_2 \leq y_1 \\ D_0(x,y) &= +\infty \text{ if } y_2 > y_1. \end{aligned}$$

$$
\begin{aligned}
D_i(y,x) &= \sup_{\lambda}\left\{\lambda : y_1 \le \frac{x}{\lambda}, y_2 \le y_1\right\} \\
&= \sup_{\lambda}\left\{\lambda : \lambda \le \frac{x}{y_1}, y_2 \le y_1\right\} \\
&= \sup_{\lambda}\left\{\lambda : \lambda \le \frac{x}{y_1} \le \frac{x}{y_2}, y_2 \le y_1\right\} \\
&= \frac{x}{y_1} \text{ if } y_2 \le y_1 \\
D_i(y,x) &= 0 \text{ if } y_2 > y_1.
\end{aligned}
$$

2. Consider

   $T_4 = \{(x_1, x_2, y_1, y_2) \ge 0_4 : y_1^\rho + y_2^\rho \le x_1^{\alpha_1} x_2^{\alpha_2}\}, \quad \rho \ge 1, 0 < \alpha_1 < 1, 0 < \alpha_2 < 1.$

   If

   $$y_1^\rho + y_2^\rho \le x_1^{\alpha_1} x_2^{\alpha_2}$$

   when is

   $$(\lambda y_1)^\rho + (\lambda y_2)^\rho \le (\lambda x_1)^{\alpha_1} (\lambda x_2)^{\alpha_2}?$$

   Answer: When $\lambda^\rho = \lambda^{\alpha_1+\alpha_2}$, i.e., when $\rho = \alpha_1 + \alpha_2$.

3. Define each of the following terms and explain how each type of efficiency is measured.

   (a) Technical Input Efficiency
   (b) Allocative Input Efficiency
   (c) Overall Input Efficiency

4. Inequality

   (a) Define each of the following terms:
      i. Input Distance Function
      ii. Cost Function
      iii. Mahler's Inequality.

   (b)

   $$D_i(y, x_1, x_2) = \frac{x_1 + x_2}{y}.$$

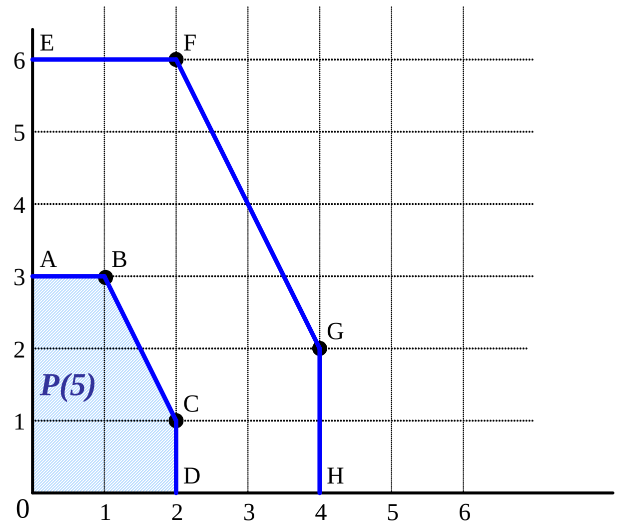
   and

   $$C(y, p_1, p_2) = \min\{p_1, p_2\} y.$$

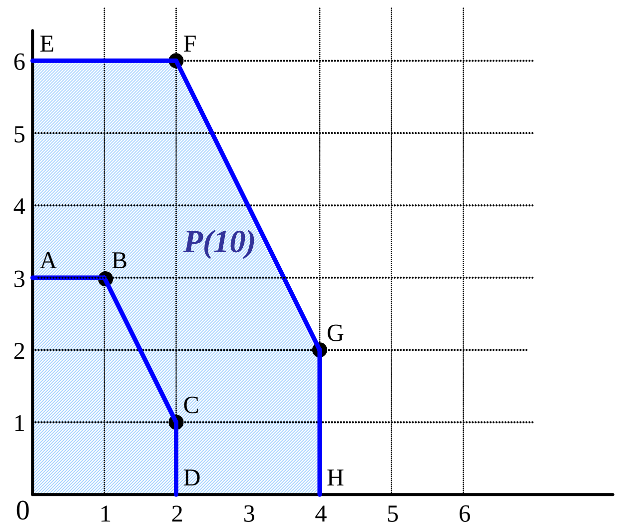
   Then

   $$
   \begin{aligned}
   [\min\{p_1, p_2\} y]\left[\frac{x_1 + x_2}{y}\right] &= [\min\{p_1, p_2\}][x_1 + x_2] \\
   &\le p_1 x_1 + p_2 x_2.
   \end{aligned}
   $$

5. The output sets are

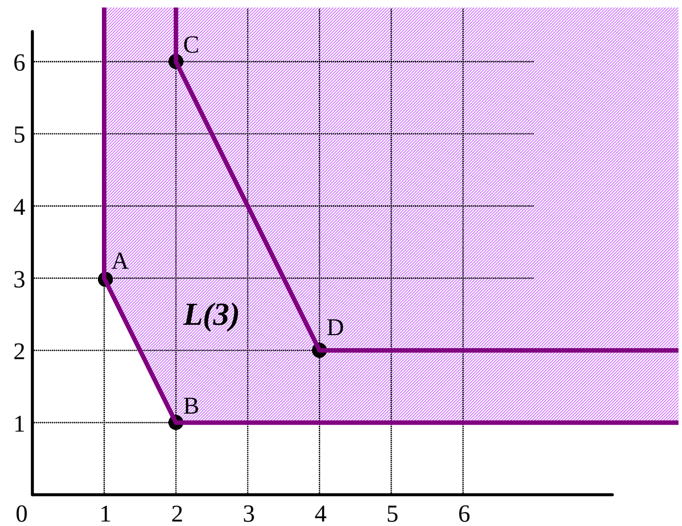


and



6. Input Sets

(a) The input sets are



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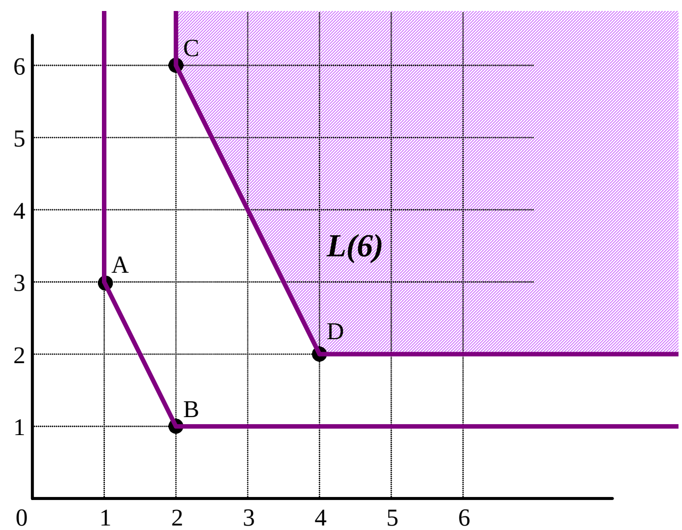
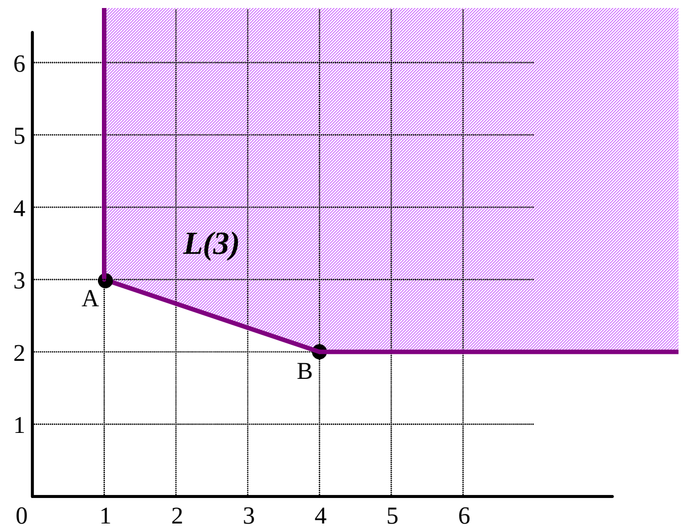


(b) The input sets are



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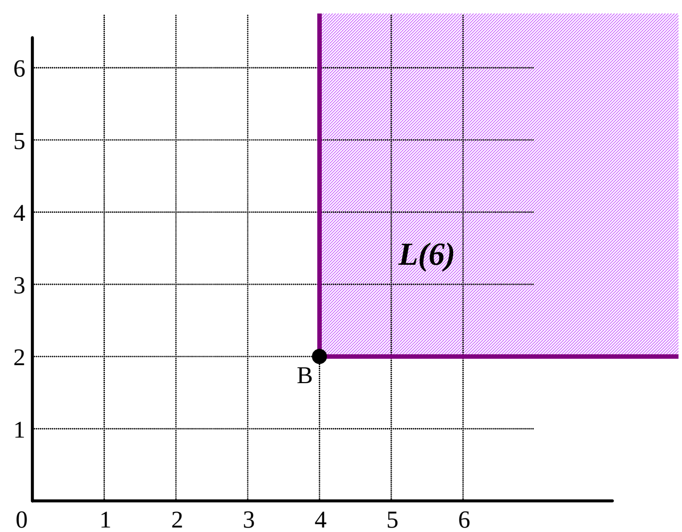

7. Given the output distance function:

$$D_o(x_1, x_2, y_1, y_2) = x_1^{-1/2} \cdot x_2^{-1/2} \cdot \max\{y_1, y_2\}.$$

the revenue function is

$$R(x_1, x_2, r_1, r_2) = (r_1 + r_2) \cdot x_1^{1/2} \cdot x_2^{1/2}.$$

8.

$$\begin{aligned} \left[(r_1 + r_2) \cdot x_1^{1/2} \cdot x_2^{1/2}\right] \left[x_1^{-1/2} \cdot x_2^{-1/2} \cdot \max\{y_1, y_2\}\right] &= (r_1 + r_2)\left(\max\{y_1, y_2\}\right) \\ &\geq r_1 y_1 + r_2 y_2 \end{aligned}$$