Economics 540B Spring 2009

Problem Set 4 Efficiency

1. Find both the output and the input distance functions for each of the following technologies.

   (a) $T_1 = \{(x_1, x_2, y) \geq 0_3 : y \leq x_1/a_1, y \leq x_2/a_2\}, \quad a_1 > 0, a_2 > 0.$

   (b) $T_2 = \{(x_1, x_2, y) \geq 0_3 : y \leq x_1^{\alpha_1} x_2^{\alpha_2}\}, \quad 0 < \alpha_1 < 1, 0 < \alpha_2 < 1.$

   (c) $T_3 = \{(x_1, x_2, y_1, y_2) \geq 0_4 :$
   $y_1/b_1 \leq x_1/a_1, y_2/b_2 \leq x_1/a_1, y_1/b_1 \leq x_2/a_2, y_2/b_2 \leq x_2/a_2\}.$

   (d) $T_4 = \{(x_1, x_2, y_1, y_2) \geq 0_4 : y_1^{\rho} + y_2^{\rho} \leq x_1^{\alpha_1} x_2^{\alpha_2}\},$
   $\rho \geq 1, 0 < \alpha_1 < 1, 0 < \alpha_2 < 1.$

   (e) $T_5 = \{(x_1, x_2, y) \geq 0_3 : y \leq a_1 x_1, y \leq b_1 x_1 + b_2 x_2\}, \quad a_1 > b_1 > 0, b_2 > 0.$

   (f) $T_6 = \{(x, y_1, y_2) \geq 0_3 : y_1 \leq x \text{ and } y_2 \leq y_1\}$

2. Give an example of a technology with at least two inputs and two outputs that exhibits constant returns to scale.

3. Define each of the following terms and explain how each type of efficiency is measured.

   (a) Technical Input Efficiency

   (b) Allocative Input Efficiency

   (c) Overall Input Efficiency

4. Inequality

   (a) Define each of the following terms:

      i. Input Distance Function

      ii. Cost Function

      iii. Mahler's Inequality.

   (b) Suppose the production function is given by $y = x_1 + x_2$. Find the input distance function and the cost function. Verify Mahler's Inequality for this example.

5. You observe two data points for a firm that produces two outputs using one input. The data are:

Obs	Input	Output 1	Output 2
1	5	1	3
2	10	4	2

Sketch the output sets $P(5)$ and $P(10)$ that correspond to the smallest technology that contains the data points and which is convex, monotonic, and exhibits constant returns to scale.

6. You observe two data points for a firm that produces one output using two inputs. The data are:

Obs	Input 1	Input 2	Output
1	1	3	3
2	4	2	6

(a) Sketch the input requirement sets $L(3)$ and $L(6)$ that correspond to the smallest technology that contains the data points and which is convex, monotonic, and exhibits constant returns to scale.

(b) Sketch the input requirement sets $L(3)$ and $L(6)$ that correspond to the smallest technology that contains the data points and which is convex, monotonic, and exhibits variable returns to scale.

7. Find the revenue function for the following output distance function.

$$D_o(x_1, x_2, y_1, y_2) = x_1^{-1/2} \cdot x_2^{-1/2} \cdot \max\{y_1, y_2\}.$$

8. A previous problem asked you to find the revenue function for the following output distance function.

$$D_o(x_1, x_2, y_1, y_2) = x_1^{-1/2} \cdot x_2^{-1/2} \cdot \max\{y_1, y_2\}.$$

Show that this example satisfies the output oriented version of Mahler's Inequality