Economics 540B Spring 2009
Problem Set 6 Consumer Theory: Choice
Short Answers

1. Marshallian demand functions must satisfy the following properties:

(a) $x_i(p,m) \geq 0, i = 1,...,k.$

$$\frac{am}{p_1} \geq 0, \frac{(1-a)m}{p_2} \geq 0$$

as long as $p_1, p_2, m \geq 0$ and $0 \leq a \leq 1$.

(b) $x_i(tp,tm) = x_i(p,m), \forall t \geq 0, i = 1,...,k.$

$$\frac{atm}{tp_1} = \frac{am}{p_1}, \frac{(1-a)tm}{tp_2} = \frac{(1-a)m}{p_2}.$$

(c) $\sum_{i=1}^{k} p_i x_i(p,m) \equiv m.$

$$\begin{aligned} p_1 \frac{am}{p_1} + p_2 \frac{(1-a)m}{p_2} &= am + (1-a)m \\ &\equiv m \end{aligned}$$

(d) The matrix of substitution terms

$$\left[\frac{\partial h_j(p,u)}{\partial p_i}\right]_{k\times k} = \left[\frac{\partial x_j(p,m)}{\partial p_i} + \frac{\partial x_j(p,m)}{\partial m} x_i(p,m)\right]_{k\times k}$$

is symmetric and negative semidefinite.

$$\begin{aligned} \left[\frac{\partial h_j(p,u)}{\partial p_i}\right]_{k\times k} &= \begin{bmatrix} -\frac{am}{p_2^2} + \frac{a^2 m}{p_2^2} & \frac{a}{p_1} \frac{(1-a)\, m}{p_2} \\ \frac{1-a}{p_2} \frac{am}{p_1} & -\frac{am}{p_1^2} + \frac{a^2 m}{p_1^2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{(a^2-a)\, m}{p_1^2} & \frac{(a-a^2)\, m}{p_1 p_2} \\ \frac{(a-a^2)\, m}{p_1 p_2} & \frac{(a^2-a)\, m}{p_2^2} \end{bmatrix}. \end{aligned}$$

The diagonal terms are negative if $0 < a < 1$ and they are nonpositive if $0 \leq a \leq 1$. The determinant is

$$\left[\frac{(a^2-a)\, m}{p_1^2}\right]\left[\frac{(a^2-a)\, m}{p_2^2}\right] - \left[\frac{(a-a^2)\, m}{p_1 p_2}\right]^2 = 0.$$

Hence, the matrix is negative semi-definite. It is also symmetric.

2. The so-called "Law of Demand" says that the demand for a good varies inversely with the price of the good, ceteris paribus. What is the relationship between the "Law of Demand" and the Slutsky equation?

$$\frac{\partial x_i(p,m)}{\partial p_i} = \frac{\partial h_i(p,u)}{\partial p_i} - \frac{\partial x_i(p,m)}{\partial m} x_i(p,m).$$

The Slutsky equation tells us that the Law of Demand will hold if

(a) good i is a normal good or if

(b) good i is an inferior good and $\frac{\partial h_i(p,u)}{\partial p_i} < \frac{\partial x_i(p,m)}{\partial m} x_i(p,m)$.

3. A first-year student once complained: "Why do we spend any time studying Hicksian demand functions? Hicksian demand functions depend on utility, which is unobservable, and thus they are also unobservable." How would you reply to this question?

The essay would include the following points. Hicksian demand function derivatives are observable via the Slutsky equation.

$$\left[\frac{\partial h_j(p,u)}{\partial p_i}\right]_{k\times k} = \left[\frac{\partial x_j(p,m)}{\partial p_i} + \frac{\partial x_j(p,m)}{\partial m} x_i(p,m)\right]_{k\times k}$$

Moreover, the properties of the Hicksian demands imply corresponding properties of the Marshallian demands. (Here, you could expand on what those properties are.)

4. GARP:

(a) If

$$p^t x^t \geq p^t x^j, p^j x^j \geq p^j x^k, ..., p^n x^n \geq p^n x^s$$

then it is *not* the case that $p^s x^s > p^s x^t$ (it is the case that $p^s x^s \leq p^s x^t$).

(b) $p^t x^t \geq p^t x^s \Rightarrow p^s x^s \leq p^s x^t$.

5. Here are two observations of a consumer's behavior.

	p_1	p_2	x_1	x_2
Obs 1	1	2	2	5
Obs 2	2	2	3	4

Is this consumer maximizing a locally nonsatiated utility function?

$$\begin{bmatrix} p^1x^1 & p^1x^2 \\ p^2x^1 & p^2x^2 \end{bmatrix} = \begin{bmatrix} 12 & 11 \\ 14 & 14 \end{bmatrix}.$$

So $p^1x^1 > p^1x^2$ and $p^2x^2 = p^2x^1$. In Varian's terminology, $x^1P^Dx^2$ and $x^2R^Dx^1 \Rightarrow x^2Rx^1$. GARP states that

$$x^2Rx^1 \Rightarrow \text{not}(x^1P^Dx^2).$$

So GARP is violated. Hence, the observed choices were not the result of maximizing a locally nonsatiated utility function.

6. Here are the combinations: yes if WARP is satisfied, no if WARP is violated.

   (a) ad: yes (Note: ad means $x^0 =$ a and $x^1 =$ d.)

   (b) ab: yes

   (c) ae: yes

   (d) bd: no

   (e) bb: yes

   (f) be: yes

   (g) cd: no

   (h) cb: no

   (i) ce: yes

7.

$$\begin{bmatrix} p^1x^1 & p^1x^2 \\ p^2x^1 & p^2x^2 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 9 & 10 \end{bmatrix}.$$

So $p^1x^1 > p^1x^2$ and $p^2x^2 > p^2x^1$. Hence, GARP is violated and consumer is not maximizing.

8.

$$\begin{bmatrix} p^1x^1 & p^1x^2 & p^1x^3 \\ p^2x^1 & p^2x^2 & p^2x^3 \\ p^3x^1 & p^3x^2 & p^3x^3 \end{bmatrix} = \begin{bmatrix} 28 & 31 & 31 \\ 31 & 28 & 31 \\ 31 & 31 & 28 \end{bmatrix}$$

In this case

$$\begin{aligned} p^1x^1 &< p^1x^2 = p^1x^3 \\ p^2x^2 &< p^2x^1 = p^2x^3 \\ p^3x^3 &< p^3x^1 = p^3x^2 \end{aligned}$$

Nothing here violates GARP so the behavior is consistent with utility maximization.