Economics 540B Spring 2009

Problem Set 7 Consumer Demand

1. A consumer's initial endowment vector is $\boldsymbol{\omega} = (\omega_1, ..., \omega_k)$. Her utility functions is Cobb-Douglas:

$$u = \prod_{i=1}^{k} x_i^{\alpha_i}, \quad \sum_{i=1}^{k} \alpha_i = 1.$$

   As usual, prices are $\mathbf{p} = (p_1, ..., p_k)$.

   (a) Show that the Marshallian demand functions are given by

$$x_i(\mathbf{p}, \mathbf{p}\boldsymbol{\omega}) = \frac{\alpha_i}{p_i}\mathbf{p}\boldsymbol{\omega}, \quad i = 1, ..., k.$$

   (b) Derive the Slutsky equation for this Cobb-Douglas consumer.

2. In the two-good case, the demand system in Question 1 is

$$x_1 = \frac{\alpha_1}{p_1}(p_1\omega_1 + p_2\omega_2) \tag{1}$$

$$x_2 = \frac{\alpha_2}{p_2}(p_1\omega_1 + p_2\omega_2) \tag{2}$$

   Find $\partial x_1/\partial p_1$. Show that $\partial x_1/\partial p_1 = 0$ when $\omega_2 = 0$.

3. Varian, Exercise 9.1, page 157. Suppose preferences are homothetic. Show that

$$\frac{\partial x_i(\mathbf{p}, m)}{\partial p_j} = \frac{\partial x_j(\mathbf{p}, m)}{\partial p_i}.$$

4. Verify that $V(P, \mathbf{q}, m)$ at the top of page 149 in Varian is an indirect utility function. In addition, verify the second equation on the same page that characterizes $X(P, \mathbf{q}, m)$.

5. For the following utility function are goods 2 and 3 separable from good 1? Justify your answer.

$$u(x_1, x_2, x_3) = x_1^{1/2}x_2^{1/4}x_3^{1/4} + x_1^{1/3}x_2^{1/3}x_3^{1/3}.$$

6. Jill likes to drink tea without cream but she also likes to drink coffee with cream. Her utility function is given by $u(x_1, x_2, z) = \min\{x_1, x_2\} \cdot z$, where x_1 is coffee, x_2 is cream, and z is tea

(a) Show that this utility function can be written in the form given by

$$u(x_1, x_2, z) = U(g(x_1, x_2), z) = U(X, z)$$

where X is an aggregate of goods x_1 and x_2.

(b) Find the demand functions for these three goods.

(c) Find the indirect utility function for this consumer.

7. There are two consumers and two goods. Let x_{ij} be the amount that consumer i consumes of good j, $\quad i, j = 1, 2$. Their utility functions and incomes are

	Consumer 1	Consumer 2
utility function	$u^1(x_{11}, x_{12}) = x_{11}^{1/3} x_{12}^{2/3}$	$u^2(x_{21}, x_{22}) = x_{21}^{2/3} x_{22}^{1/3}$
income	m_1	m_2

Find the aggregate demand for each of the two goods. Does aggregate demand depend on aggregate income?