Economics 540B Spring 2009
Problem Set 8 Uncertainty

1. Mary Jane has a utility function of wealth given by $u(w) = \sqrt{w}$, where w is wealth. Her initial wealth is \$100. She also has a lottery ticket that will be worth \$44 with a probability of 1/2 and will be worth \$0 with a probability of 1/2.

   (a) What is her expected utility?

   (b) What is the lowest price p at which she would part with the lottery ticket?

2. In a small town near SIU Carbondale there is a probability p that there will be a flood that will cause L dollars worth of damage to a typical homeowner. The homeowner is able to buy flood insurance; the amount of flood insurance is q dollars and each unit (dollar) of insurance costs π where π is the insurance premium per dollar of coverage. Thus, the total insurance premium paid is πq. The insurance is actuarially fair so that $\pi = p$. The homeowner's initial wealth is W and the homeowner is an expected-utility maximizer.

   (a) How much insurance will the homeowner purchase?

   (b) Now suppose that homeowners have learned over time that when there is a flood, the federal government steps in and offers grants to rebuild the damaged homes. Suppose further that these grants offset a fraction, t, of the total loss L leaving the homeowner to pay for the remaining loss, $(1-t)L$. In this case, how much insurance will the homeowner purchase?

3. Suppose you currently have a job in city A that pays \$1600 per week. If you move to city B and search for a job there is a probability of 1/2 that you will get a job that pays \$2500 per week and a probability of 1/2 that you will get a job that pays \$900 per week. Your utility depends only on your weekly salary . (I.e., except for the salaries, the jobs and the cities are equally attractive and moving costs are zero).

   (a) Which option would you choose, i.e. would you stay in city A at your current job or would you move to city B and search for a job?

   (b) A friend named Sue has a utility function given by $u(x) = \sqrt{x}$ where x is her monthly salary. She faces the same options that you do. Which of the above options would Sue choose?

   (c) Given your answers to 3a and 3b, who is more risk averse, you or Sue?

   (d) Another friend named George has a utility function given by $u(x) = \ln x$ where x is his monthly salary. He also faces the same options that you and Sue do. Who is more risk averse, Sue or George? Which of the above options would George choose?

4. A risky alternative pays \$4 with probability 1/2 and pays \$16 with probability 1/2. Find the certainty equivalent of this risky alternative for the Bernoulli utility function, $u(x) = x^{1/2}$.

5. The Arrow-Pratt measure of *absolute* risk aversion is

$$r(w) = -\frac{u''(w)}{u'(w)}.$$

What is the form of the expected utility function if $r(w) = r =$ a constant.

6. The Arrow-Pratt measure of *relative* risk aversion is

$$\rho(w) = -\frac{u''(w)w}{u'(w)}.$$

What is the form of the expected utility function if $\rho(w) = \rho =$ a constant.

7. A consumer has an expected utility function given by $u(w) = w^{1/2}$. She is given the choice of a lottery:

$$L = p \circ x \oplus (1-p) \circ y$$

where $x = \$10,000$ and $y = \$0$ or a sure thing, namely $w = \$6400$. Find the value of the probability, p, that makes her indifferent between the lottery L and the sure thing.

8. The best lottery, B, is a sure wealth of \$100 and the worst lottery, W, is a sure wealth of \$0. The expected utility function is $u(w) = w^{1/2}$. Find another expected utility, $v(w)$, that represents the same preferences over lotteries and that has the property that $v(W) = 0$ and $v(B) = 1$.

9. John's preference ordering over lotteries satisfy the assumption of the expected utility theorem. When confronted with a choice between the following two lotteries, A and B, he prefers A over B.

$$A = \left\{ \begin{array}{ccc} \$1 \text{ million} & \text{with } p = 1 \end{array} \right. \qquad B = \left\{ \begin{array}{ccc} \$3 \text{ million} & \text{with } p = .1 \\ \$1 \text{ million} & \text{with } p = .8 \\ \$0 & \text{with } p = .1 \end{array} \right.$$

Suppose that John is now given the choice between the following two lotteries, C and D.

$$C = \left\{ \begin{array}{ccc} \$1 \text{ million} & \text{with } p = .2 \\ \$0 & \text{with } p = .8 \end{array} \right. \quad D = \left\{ \begin{array}{ccc} \$3 \text{ million} & \text{with } p = .1 \\ \$0 & \text{with } p = .9 \end{array} \right.$$

Which of these two lotteries does John prefer?