Economics 540B — Spring 2009

Problem Set 9 General Equilibrium

1. In a two-consumer, two-good pure exchange economy the two utility functions and initial endowments are given by

$$u_1(x_1^1, x_1^2) = \left(x_1^1\right)^a \left(x_1^2\right)^{1-a} \qquad \boldsymbol{\omega}_1 = \left(\omega_1^1, \omega_1^2\right)$$

$$u_2(x_2^1, x_2^2) = \left(x_2^1\right)^b \left(x_2^2\right)^{1-b} \qquad \boldsymbol{\omega}_2 = \left(\omega_2^1, \omega_2^2\right)$$

where $0 < a < 1, 0 < b < 1$.

   (a) Show that the demand functions are given by

$$x_1^1 = \frac{a\left(p_1\omega_1^1 + p_2\omega_1^2\right)}{p_1}, \quad x_1^2 = \frac{(1-a)\left(p_1\omega_1^1 + p_2\omega_1^2\right)}{p_2},$$

$$x_2^1 = \frac{b\left(p_1\omega_2^1 + p_2\omega_2^2\right)}{p_1}, \quad x_2^2 = \frac{(1-b)\left(p_1\omega_2^1 + p_2\omega_2^2\right)}{p_2}.$$

   (b) Find the aggregate excess demand functions, $z_1(\mathbf{p})$ and $z_2(\mathbf{p})$.

   (c) Using your answer from part (b), verify that Walras's Law holds for these Cobb-Douglas consumers.

   (d) Find the Walrasian equilibrium price ratio, $p = p_1/p_2$.

2. In a two-consumer, two-good pure exchange economy the two utility functions and initial endowments are given by

$$u_1(x_1^1, x_1^2) = \left(x_1^1\right)^a \left(x_1^2\right)^{1-a} \qquad \boldsymbol{\omega}_1 = \left(\omega^1, 0\right)$$

$$u_2(x_2^1, x_2^2) = \left(x_2^1\right)^b \left(x_2^2\right)^{1-b} \qquad \boldsymbol{\omega}_2 = \left(0, \omega^2\right)$$

where $0 < a < 1, 0 < b < 1$. Find the Walrasian equilibrium price ratio, $p = p_1/p_2$, and the Walrasian equilibrium quantities, $(x_1^1, x_1^2, x_2^1, x_2^2)$.

3. (Varian, Exercise 17.4) There are two consumers A and B with the following utility functions and endowments:

$$u_A\left(x_A^1, x_A^2\right) = a\ln x_A^1 + (1-a)\ln x_A^2 \quad \boldsymbol{\omega}_A = (0, 1)$$

$$u_B\left(x_B^1, x_B^2\right) = \min\left(x_B^1, x_B^2\right) \quad \boldsymbol{\omega}_B = (1, 0)$$

where $0 < a < 1$. Calculate the market clearing prices and the equilibrium allocation.

4. There are two consumers, A and B with the following utility functions and endowments:

$$\begin{aligned} u_A\left(x_A^1, x_A^2\right) &= a \ln x_A^1 + (1-a) \ln x_A^2 \quad \boldsymbol{\omega}_A = (0,1) \\ u_B\left(x_B^1, x_B^2\right) &= x_B^1 + x_B^2 \quad \boldsymbol{\omega}_B = (1,0) \end{aligned}$$

where $0 < a < 1$. Calculate the market clearing prices and the equilibrium allocation.

5. There are two consumers, A and B with the following utility functions and endowments:

$$\begin{aligned} u_A\left(x_A^1, x_A^2\right) &= \min\left(x_A^1, x_A^2\right) \quad \boldsymbol{\omega}_A = (0,3) \\ u_B\left(x_B^1, x_B^2\right) &= x_B^1 + 2x_B^2 \quad \boldsymbol{\omega}_B = (3,0) \end{aligned}$$

Calculate the market clearing prices and the equilibrium allocation.

6. A pure exchange two-good economy consists of n identical consumers with utility functions given by

$$u(x_i^1, x_i^2) = \left(x_i^1\right)^2 + \left(x_i^2\right)^2, \quad i = 1, ..., n.$$

Each consumer has an initial endowment given by

$$(\omega_i^1, \omega_i^2) = (1,1).$$

   (a) Show that the demand function for each good for each consumer is discontinuous at $p_1/p_2 = 1$. (They also fail to be single-valued functions at $p_1/p_2 = 1$.)

   (b) Suppose n is an even number. Find the competitive equilibrium. (Hint: Start with the case $n = 2$. Draw the Edgeworth box and find the competitive equilibrium. Then generalize your result to $n = 2z$ where z is a positive integer.

   (c) Suppose n is an odd number. Show that a competitive equilibrium does not exist. (Hint: Begin with $n = 2z$ where z is a positive integer and then add one more consumer so that $n = 2z + 1$. What happens?)

7. (Varian, Problem 17.2) Draw an Edgeworth box example with an infinite number of (relative) prices that are Walrasian equilibria.

8. You are the chief of a tribe of 100 individuals. Your loyal subjects have harvested the only two goods that are consumed in your society. After giving you a tribute of each good the remaining quantities are 500 coconuts (for food) and 300 yards of fiber (for clothing). As a wise and benevolent chief how will you allocate these two goods among your subjects? Justify your answers. The more answers that you can justify the more interesting will be your response.

9. PURE EXCHANGE ECONOMY:

(a) For a pure exchange economy, define the following terms: feasible allocation, weakly Pareto efficient, strongly Pareto efficient, contract curve, Walrasian equilibrium, and aggregate excess demand. State Walras' Law. Under what assumption does Walras' Law hold?

(b) Suppose there are two consumers with utility functions

$$u_1 = x_{11}^{1/3} x_{12}^{2/3} \text{ and } u_2 = \min\{x_{21}, x_{22}\}$$

and initial allocations

$$(\omega_{11}, \omega_{12}) = (12, 0) \text{ and } (\omega_{21}, \omega_{22}) = (0, 24).$$

i. Find the contract curve.
ii. Find the equilibrium price ratio p_1/p_2.
iii. Find the four equilibrium quantities, $x_{11}, x_{12}, x_{21}, x_{22}$.

10. EQUILIBRIUM WITH PRODUCTION:

(a) Each consumer in a small closed economy has a utility function

$$u(b, g) = \sqrt{b} + \sqrt{g}$$

where b is butter and g is guns. Each consumer also supplies one unit of labor for which they receive a labor wage equal to w. Set up the utility maximization problem and show that the Marshallian demands are given by

$$b = \frac{p_g}{p_b(p_b + p_g)} w \text{ and } g = \frac{p_b}{p_g(p_b + p_g)} w.$$

(b) Each firm in this perfectly competitive economy can produce both butter and guns. The production functions are

$$b = 4L_b \text{ and } g = 2L_g,$$

where L_b and L_g are the quantities of labor allocated to butter and guns, respectively. We will assume, without loss of generality, that there is only one competitive firm. Why is this assumption justified?

(c) Suppose there are 120 consumers. Find the competitive equilibrium price ratios. Find the total quantity of butter and guns produced. How much labor is allocated to each output?