Economics 540B Spring 2010

Problem Set 1: Technology and Profit

1. (Varian, Problem 1.10) The technology Y is *additive* if

$$\mathbf{y} \in Y \text{ and } \mathbf{y}' \in Y \Rightarrow \mathbf{y} + \mathbf{y}' \in Y.$$

The technology is *divisible* if

$$\mathbf{y} \in Y \text{ and } 0 \leq t \leq 1 \Rightarrow t\mathbf{y} \in Y.$$

Show that if a technology is both additive and divisible, then Y must be convex and exhibit constant returns to scale.

2. A production set, denoted by $Y \subseteq R^n$, is the set of feasible production, or netput, vectors. Define the following terms: a) Y is *additive*, b) Y exhibits *Nonincreasing Returns to Scale (NIRS)*, c) Y is a *convex cone*. What is the relationship between these three concepts?

3. Consider the production function

$$y = Ax_1^a x_2^b, \qquad a > 0, b > 0, a + b < 1.$$

   (a) Calculate the elasticity of scale.

   (b) Add a third input, z, in such a way that the elasticity of scale is 1. How is this procedure justified?

4. A simple candidate for a production function with one input is: $y = \ln x$.

   (a) Why is this a bad choice?

   (b) Consider a better candidate given by: $y = \ln(x + 1)$. Why is this a better choice?

   (c) Convert y and x into netputs y_2 and y_1, respectively, and specify the netput set Y in such a way that Y satisfies netput monotonicity.

5. Find the profit function for the technology given by $Y = \{(y_1, y_2) : y_1 + y_2^2 \leqq 0\}$.

6. State the *Weak Axiom of Profit Maximization*. Now consider the following data for a competitive firm. There are two observations of the firm's vectors of netputs and their corresponding prices.

Period	p_1	p_2	y_1	y_2
1	2	8	-4	2
2	6	12	-1	1

Is this data consistent with profit-maximizing behavior?

7. Show that if the technology Y exhibits nondecreasing returns to scale then $\pi(\mathbf{p}) = 0$ or $\pi(\mathbf{p}) = +\infty$.

8. Suppose the Weak Axiom of Profit Maximization (WAPM) is not satisfied for a set of data given by $(\mathbf{p}^t, \mathbf{y}^t), t = 1, ..., T$. Why might WAPM fail? List as many reasons as possible and briefly justify each reason.

9. Given a technology set Y, the set of all feasible netput vectors, that is not directly observable, one can attempt to find inner and outer approximations to Y using an appropriate set of data.

   (a) What is meant by the term "inner approximation"? What assumptions must be made to ensure that such an approximation can be made? What data are needed?

   (b) What is meant by the term "outer approximation"? What assumptions must be made to ensure that such an approximation can be made? What data are needed?

10. Consider the production function: $y = x + x^{1/2}$.

    (a) Show that this production function is strictly concave.

    (b) Find the profit function. (Let p be the output price and w the input price.) Be sure to determine the range of values of p and w for which the profit function is well-defined.

11. (Varian, 3.2, p. 48) Consider the technology described by $y = 0$ for $x \leqq 1$ and $y = \ln x$ for $x > 1$. Calculate the profit function for this technology.