

Problem Set 2 Cost and Duality

1. A firm maximizes profit over the technology set

$$T = \{(\mathbf{x}, \mathbf{y}) : \mathbf{x} \in R_+^N, \mathbf{y} \in R_+^M, \mathbf{x} \text{ can produce } \mathbf{y}\},$$

by choosing $(\mathbf{x}^*, \mathbf{y}^*)$ when the corresponding price vectors are (\mathbf{w}, \mathbf{p}) . Show that \mathbf{x}^* is the cost-minimizing choice when producing \mathbf{y}^* at input prices, \mathbf{w} .

2. [Varian, Problem 4.3, page 63] A firm has two plants with cost functions $c_1(y_1) = y_1^2/2$ and $c_2(y_2) = y_2$. Total output is $y = y_1 + y_2$. Find the cost function, $c(y)$, for the firm.
3. [Varian, Problem 5.6, page 78] Consider a firm with conditional factor demand functions of the form

$$\begin{aligned} x_1 &= 1 + 3w_1^{-1/2}w_2^a \\ x_2 &= 1 + bw_1^{1/2}w_2^c. \end{aligned}$$

Output has been set equal to 1 for convenience. What are the values of the parameters a, b , and c and why?

4. The following table shows two observations on factor demand, x_1, x_2 , factor prices, w_1, w_2 , and scalar output, y , for a firm. Is the data in this table consistent with cost-minimizing behavior?

Obs	y	w_1	w_2	x_1	x_2
A	10	3	2	20	20
B	10	4	1	25	15

Be sure to clearly justify your answer!

5. You are given the following data for a competitive firm.

Obs	p	w_1	w_2	y	x_1	x_2
1	6	1	4	100	20	60
2	5	2	3	80	40	30

- (a) Is this data consistent with the hypothesis of cost minimization? Explain.
- (b) Is this data consistent with the hypothesis of profit maximization? Explain.

6. Prove directly, without taking derivatives, that the cost function cannot be strictly concave.
7. The cost function is defined by

$$c(\mathbf{w}, y) = \min_{\mathbf{x}} \{\mathbf{w}\mathbf{x} : \mathbf{x} \in V(y)\}$$

where $V(y)$ is the input requirement set.

- (a) When is it possible to exactly construct $V(y)$ given the cost function, $c(\mathbf{w}, y)$? How is it done?
- (b) Regardless of whether one can exactly construct $V(y)$ from the cost function, show that we can always construct a set, $V^*(y)$, such that $V(y) \subseteq V^*(y)$.
- (c) Here is a quote from your textbook. “*the cost function of a firm summarizes all of the economically relevant aspects of its technology.*” Varian (1992, page 84) Explain what Varian means by this statement. Part of your answer should include an explanation of the term “*economically relevant aspects of its technology*”. Be sure to justify the entire quoted statement.
8. A firm’s cost function is given by

$$c(\mathbf{w}, y) = \gamma_1 w_1 + 2w_1^{1/2}w_2^{1/2}y + \gamma_2 w_2, \quad \gamma_1 > 0, \gamma_2 > 0.$$

- (a) Show that this cost function is monotonic and concave in input prices.
- (b) Find the firm’s production function.
9. Find the input requirement set, $V(y)$, for each of the following cost functions. You should show all of your work and justify each step in your reasoning.

(a) $c(\mathbf{w}, y) = \min \left\{ \frac{w_1}{a_1}, \frac{w_2}{a_2} \right\} y$

(b) $c(\mathbf{w}, y) = (b_1 w_1 + b_2 w_2) y$