

## The Work of W. M. Gorman

by

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### 1. Introduction.

W. M. Gorman lived from 1923 to 2003. He graduated from Trinity College, Dublin, in 1948 in economics and a year later in mathematics. He held posts at the University of Birmingham, Oxford, and the London School of Economics. Among many honours, he was the president of the Econometric Society in 1972, a fellow of the British Academy, and an honorary foreign member of the American Academy of Arts and Sciences and of the American Economic Association.<sup>1</sup>

Gorman was interested in economics interpreted very broadly. He read history, philosophy, mathematics, and statistics. Nevertheless, most of his work was very abstract and many readers found it difficult to follow. We begin here by discussing his long-standing interest in two kinds of aggregation, over agents and over commodities, before considering some of his other contributions.

### 2. Aggregation Across Agents

Gorman's first published paper (Gorman [1953]) provided necessary and sufficient conditions for the existence of a representative consumer. That is, given that each consumer in society has well-behaved preferences and demand functions,

$$x^h = f^h(p, m_h) \quad \text{for } h = 1, \dots, H, \quad (2.1)$$

when can aggregate demand,  $\sum_h x^h$ , be generated by the demands of an aggregate agent so that

$$x = \sum_h x^h = f(p, m), \quad (2.2)$$

where  $m = \sum_h m_h$ ? He demonstrated that a necessary and sufficient condition is that consumers have affine parallel Engel curves,

$$x^h = a(p)m_h + b^h(p) \quad (2.3)$$

so that the demands of the aggregate consumer can be written as

$$x = a(p)m + b(p), \quad (2.4)$$

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<sup>1</sup> For a more detailed discussion of Gorman's career and work, see Honohan and Neary [2003].

where  $b(p) = \sum_h b^h(p)$ . In a later paper, Gorman [1961] provided a closed form representation of the preferences of consumer with parallel affine Engle curves: the indirect utility functions can be written as

$$V^h(p, m_h) = A(p)m_h + B^h(p) \quad \text{for } h = 1, \dots, H, \quad (2.5)$$

and the indirect utility function of the representative consumer can be written as

$$V(p, m) = A(p)m + B(p), \quad (2.6)$$

where  $B(p) = \sum_h B^h(p)$ . Using Roy's theorem the reader can easily check that (2.3) and (2.4) are derived from (2.5) and (2.6) respectively. The intuition behind this result is straightforward: if we take a dollar away from consumer one and give it to consumer two, aggregate demand cannot change, because total income has not changed; the two consumers therefore must have identical marginal propensities to consume.

There have been many attempts to generalize this notion of a representative agent,<sup>2</sup> but all suggest that aggregate demand for the  $i$ th commodity can be written as

$$x_i = \sum_{j=1}^J a_{ij}(p)b_j(m, z), \quad (2.7)$$

where  $z$  is a vector of characteristics. Gorman [1981] demonstrates that utility maximization requires that the rank of the matrix with elements  $a_{ij}$  be less than or equal to three. Note that the idea of a representative consumer given in (2.3) entails that the matrix have rank two. This proved that there is not much more that could be done to generalize this idea.

Over the course of the sixties a battle raged between the U.S. Cambridge and the English Cambridge over—to put it crudely—whether the marginal product of capital was a meaningful aggregate concept.<sup>3</sup> Gorman [1968a] approached this problem in the dual, using profit functions. His argument in favor of using the dual was that it was important to model problems in the appropriate variables. When thinking about aggregation he noted that the individual agents all face the same prices. Hence he thought the problem should be attacked using profit rather than production functions.

In the simplest terms, suppose that each firm has a profit function given by

$$\pi^f(p, k^f) = \max_{y^f} \{p \cdot y^f \mid y^f \in T^f(k^f)\} \quad \text{for } f = 1, \dots, F, \quad (2.8)$$

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<sup>2</sup> See, *e.g.*, Muellbauer [1976].

<sup>3</sup> See Fisher and Monz [1992] for an extensive discussion of this issue.

where  $k^f$  is a vector of the quasi-fixed factors of firm  $f$  and  $T^f(k^f)$  is the technology set of the firm for each fixed value of the  $k^f$ . In this context, the marginal product of capital only makes sense if there exists an aggregate profit function that can be written as

$$\pi(p, k) = \max_y \{p \cdot y \mid y \in T(k)\}, \quad (2.9)$$

where  $k$  is a scalar measure of the aggregate capital stock. Gorman showed that a necessary and sufficient condition for (2.9) is that

$$\pi^f(p, k^f) = \alpha(p)\phi^f(k^f) + \beta^f(p) \quad \text{for } f = 1, \dots, F, \quad (2.10)$$

so that the aggregate capital stock is given by

$$k = \sum_f \phi^f(k^f), \quad (2.11)$$

thus demonstrating that the marginal product of capital is not likely to be a meaningful concept very often.<sup>4</sup>

The intuition for this result is a little less straightforward. Think of  $\phi^f(k^f)$  as a measure of bolted-down capital of firm  $f$ . Then,  $\alpha(p)$  is clearly the marginal product of capital in firm  $f$  and (2.10) requires this be the same in every firm so that the aggregate profit function can be written as

$$\pi(p, k) = \alpha(p)k + \beta(p) \quad (2.12)$$

where  $k$  is given by (2.11) and  $\beta(p) = \sum_f \beta^f(p)$ .

### 3. Aggregation Across Commodities.

Gorman's best-known paper in this area, Gorman [1959a], was written in response to a paper by Strotz [1957] on two-stage budgeting. Suppose that a consumer (or any organization with a well-behaved objective function and an expenditure constraint) wants to simplify its purchasing decisions as follows: first it wants to allocate funds optimally to broad categories of commodities and then later make the detailed calculations of how to spend the funds within any particular category. For example, a consumer might first decide how much money to budget for food and then later decide exactly how to allocate the food budget to particular commodities. The latter decision turns out to be equivalent to the separability of the commodities in each category, whereas the former hinges on a notion of price aggregation, both of which are discussed elsewhere in this volume.<sup>5</sup>

<sup>4</sup> For the implied restrictions on the technology sets and production functions, see Blackorby and Schworm [1984].

<sup>5</sup> For more on two-stage budgeting and separability, see *Separability* by Blackorby, Primont, and Russell [this volume].

Separability seemed a natural assumption to Gorman; it allowed the researcher to focus on a particular group of commodities without having to worry very much about anything else. In Gorman [1968b], however, he showed that this could have previously unknown implications. Suppose that one assumed that two groups of commodities were separable from their complements so that the utility function could be written as

$$U(x) = \bar{U} \left( U^A(x^A), x^C \right) \quad \text{and} \quad U(x) = \hat{U} \left( U^B(x^B), x^D \right). \quad (3.1)$$

Suppose in addition that some of the commodities in group A are not in group B, some are in both A and B, and some are in B but not in A. Gorman showed that this implied that the utility function could in fact be written as

$$U(x) = \check{U}^* \left( U^a(x^a) + U^b(x^b) + U^c(x^c) \right), \quad (3.2)$$

where the variables in  $a$  are those that are in A and not in B, those in  $b$  are in B but not in A, and those in  $c$  are in both groups. This surprising result can be taken in two ways: (1) perhaps too much separability is a dangerous thing but (2) on the other hand this theorem provides a fundamental and clean way of modeling additivity.

Ever attune to different ways of looking at things, Gorman [1970, 1995], in an unpublished paper, noted that if a quasi-concave utility function could be written additively, as would often be required to study intertemporal or uncertain events,

$$U(x) = \check{U}^* \left( \sum_{s=1}^S U^s(x^s) \right), \quad (3.3)$$

then at most one of the functions  $U^s$  could fail to be concave. Thus, for example, if the function  $U^s$  functions were the same as in expected utility theory (*i.e.*, linear transformations of one another), then quasi-concavity would imply concavity. Again this can be taken two ways—as an easy way to obtain concavity or as the danger of too much separability.

#### 4. General Interests.

Gorman was well known for having widely circulated and widely cited but unpublished papers. Perhaps, the best known of these was “A Possible Procedure for Analysing Quality Differentials in the Egg Market”, a 1956 Working Paper of the Iowa Agricultural Experiment Station.<sup>6</sup> Suppose that it is not the commodities that consumers want but rather the characteristics embodied in them. Thus type  $i$  egg would contain  $a_i$  of characteristic A,  $b_i$  of characteristic B, and so on. Further, he assumed that if one bought  $x_1$  of type one egg and  $x_2$  of type two then the total amount of characteristic A would be  $a_1x_1 + a_2x_2$ .

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<sup>6</sup> This paper was eventually published as Gorman [1980]. A more complete discussion can be found in Honohan and Neary [2003].

Thus, at arbitrary prices only two types of eggs would be purchased if there were only two characteristics that were relevant and three types if three characteristics were relevant, except in the degenerate case where relative prices were just right. Then the consumer would be indifferent between two or three or more types of eggs. Gorman argued that equilibrium prices should not contain any arbitrage opportunities and hence the degenerate case would be the normal one. This then suggested an agenda for empirical work that he and students pursued over the years.<sup>7</sup>

Although Gorman wrote and published little in welfare economics, what he did write demonstrated a profound understanding of the issues involved. In order to avoid the known problem with the Kaldor [1939] compensation principle (namely that situation B could be preferred to situation A by the Kaldor compensation criterion and that A could be preferred to B by the same criterion), Scitovsky [1941] had proposed a new test that required that B should be preferred A if B was preferred by the Kaldor compensation test and A was not preferred to B by the same test. In Gorman [1955] he demonstrates in a very elegant manner that the Scitovsky criterion is intransitive; that is, that B could be preferred to A by the Scitovsky criterion, C preferred to B by the same criterion, and then that A could be preferred to C by the same test.

In a rather more philosophical vein, Gorman [1959b] presents a series of arguments that might lead one to think that social indifference curves are convex. In this he was clearly aware of the problems of interpersonal comparisons of utility and the idea of diminishing marginal utility for each individual. Even now this makes for a thought-provoking read.

Although Gorman did very little research in general equilibrium, when preparing a paper for a festschrift in honor of Ivor Pierce he felt that only a general equilibrium paper would be appropriate. Gorman [1984] prepared an analysis of the Le Chatelier principle in general equilibrium. To give the reader the flavour of his elegant argument, let  $\Pi(p)$  and  $\pi(p)$  be the long-run and short-run profit functions of a firm. It seems natural to assume that

$$\Pi(p) \geq \pi(p). \quad (4.1)$$

Suppose that at  $p = \bar{p}$  we are at a long-run equilibrium, so that

$$\Pi(\bar{p}) = \pi(\bar{p}). \quad (4.2)$$

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<sup>7</sup> Not very much of this research actually surfaced, although a 1959 University of Birmingham working paper entitled “Demand for Fish: an Application of Factor Analysis” and his 1972 presidential address to the Econometric Society, “A Sketch for the Demand for Related Goods”, were widely cited for some time.

Thus, the price vector  $p = \bar{p}$  minimizes the difference between long-run and short-run profits,  $\Pi(p) - \pi(p)$ . The second-order condition for this minimization problem is given by

$$\sum_i \sum_j \Pi_{ij}(\bar{p})\theta_i\theta_j \geq \sum_i \sum_j \pi_{ij}(\bar{p})\theta_i\theta_j. \quad (4.3)$$

By Hotelling's lemma, this implies immediately that

$$\frac{\partial \bar{X}_i}{\partial p_i} \geq \frac{\partial \bar{x}_i}{\partial p_i}; \quad (4.4)$$

that is, the long-run response of commodity  $i$  to a change in its price is great than or equal to the short-run response—the Le Chatelier principle.

Technically, much of Gorman's work was difficult; he frequently employed transformations of variables and functions with such speed that the reader felt, if not lost, at least dizzy. In Gorman [1976] he wrote what he thought of as a reasonable introduction to some of these tools and called the paper "Tricks with Utility Functions". We conclude with a discussion of one of these tricks and the lesson that Gorman thought could be learned from it.

Gorman begins his discussion of equivalent adult scales with a quote from a former schoolmaster who said "When you have a wife and a baby, a penny bun costs threepence". Consider a family of type  $a = (a_1, \dots, a_n)$  whose utility function could be written—after Barten [1964]—as

$$u_a = U^a(x) = U(x^a), \quad (4.5)$$

where the adjusted consumption vector,

$$x^a = \left( \frac{x_1}{a_1}, \dots, \frac{x_N}{a_N} \right), \quad (4.6)$$

corrects for the number of equivalent adults. Note that the second equal sign implies that all households have the same utility function but defined on the adjusted variables. The expenditure function dual to  $U^a$  in (4.5) is given by

$$\begin{aligned} E^a(u_a, p) &= \min_x \{p_1x_1 + p_2x_2 + \dots + p_Nx_N \mid U^a(x) \geq u_a\} \\ &= \min_x \{a_1p_1x_1^a + \dots + a_Np_Nx_N^a \mid U(x^a) \geq u_a\} \\ &= E(u_a, p^a), \end{aligned} \quad (4.7)$$

where  $p^a = (a_1p_1, \dots, a_Np_N)$  and  $E$  is the expenditure function dual to  $U$  in (4.5). Thus, for a family of three bread-equivalent adults "a penny bun costs threepence". The adjusted compensated demand for good  $i$  is

$$x_i^a = E_i(u_a, p^a), \quad (4.8)$$

so that the ordinary compensated demand is

$$x_i = a_i E_i(u_a, p^a). \quad (4.9)$$

From this, the compensated demand elasticities can be written as

$$\epsilon_{ij} = \delta_{ij} + \alpha_{ij}, \quad (4.10)$$

where  $\delta_{ij}$  is the Kronecker delta and

$$\alpha_{ij} = \frac{\partial \ln x_i}{\partial \ln a_j} \quad (4.11)$$

at a fixed  $u_a$  is the compensated elasticity with respect to family size. It is easy from here to calculate the uncompensated elasticities as well. From this Gorman concludes that “Were the theory true, and were the sample to include a great enough variety of family types, we could use them to calculate the price elasticities from survey data. As long as everyone faces the same prices, we need not even know what they are.” If one had tried to do this analysis, as Barten did, working only with (4.5) the simplicity of this model would not have been exposed.

This paper had dozens of such “tricks,” and we recommend them to the reader.

## References

Barten, A., “Family Composition, Prices, and Expenditure Patterns” in *Econometric Analysis for National Economic Planning*, edited by P. E. Hart, G. Mills and J. K. Whitaker, Butterworths, 1964.

Blackorby, C., and B. Schworm, “The Structure of Economies with Aggregate Measures of Capital: A Complete Characterization”, *Review of Economic Studies* **51**, 1984, 633–650.

Fisher, F. M., and J. Monz, *Aggregate Production Functions and Related Topics*, The MIT Press: Cambridge MA, 1992.

Gorman, W. M., “Community Preference Fields”, *Econometrica* **51**, 1953, 63–80.

Gorman, W. M., “The Intransitivity of Certain Criteria Used in Welfare Economics”, *Oxford Economic Papers* **11**, 1955, 23–55.

Gorman, W. M., “Separability and Aggregation”, *Econometrica* **27**, 1959a, 469–481.

Gorman, W. M., “Are Social Indifference Curves Convex?”, *Quarterly Journal of Economics* **73**, 1959b, 485–496.

Gorman, W. M., “On a Class of Preference Fields”, *Metroeconomica* **13**, 1961, 53–56.

Gorman, W. M., “Measuring the Quantities of Fixed Factors”, in *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Edinburgh, Edinburgh University Press, 1968a, 141–172.

Gorman, W. M., “The Structure of Utility Functions”, *Review of Economic Studies* **32**, 1968b, 369–390.

Gorman, W. M., “The Concavity of Additive Utility Functions”, typescript, University of North Carolina, 1970, in Gorman [1995], 127–136.

Gorman, W. M., “Tricks with Utility Functions”, in *Essays in Economic Analysis*, edited by M. Artis and R. Nobay, Cambridge University Press, Cambridge, 1976.

Gorman, W. M., “The Demand for Related Goods: A Possible Procedure for Analysing Quality Differentials in the Egg Market”, *Review of Economic Studies* **47**, 1980, 843–856.

Gorman, W. M., “Some Engel Curves”, in *Essays in the Theory and Measurement of Demand in Honour of Sir Richard Stone*, edited by A. Deaton, Cambridge, Cambridge University Press, 1981.

Gorman, W. M., “Le Chatelier and General Equilibrium”, in *Demand, Equilibrium and Trade: Essays in Honour of Ivor F. Pearce*, edited by A. Ingham and A. M. Ulph, London, Macmillan, 1984, 1–18.

Gorman, W. M., *The Collected Works of W. M. Gorman, Vol. I*, edited by C. Blackorby and A. Shorrocks, Oxford University Press, Oxford, 1995.

Honohan, P., and J. P. Neary, “W. M. Gorman”, *The Economic and Social Review* **34**, 2003, 195–209.

Kaldor, N. “Welfare Propositions of Economics and Interpersonal Comparisons of Utility”, *Economic Journal* **49**, 1939, 549–552.

Muellerbauer, J., “Community Preference Fields and the Representative Consumer”, *Econometrica* **44**, 1976, 979–999.

Scitovsky, T., “A Note on Welfare Propositions in Economics”, *Review of Economic Studies* **9**, 1941, 77–88.

Strotz, R., “The Empirical Implications of a Utility Tree”, *Econometrica* **25**, 1957, 269–280.