An Efficient Technique For Multi-Phase Model Based Iterative Reconstruction

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Abstract—Multi-phase scan is a fundamental CT acquisition technology used in a variety of applications such as cardiac CT and perfusion CT. Model-based iterative reconstruction (MBIR) has already demonstrated significant IQ improvements that can provide significant noise reduction with improved resolution that are essential for such applications. However the challenge is to efficiently reconstruct multi-phase scans. The simplest way is to perform MBIR reconstruction on each individual phase data in a sequential order or in a parallel computing framework. However, these approaches either lead to increase in overall compute time or reduce the patient throughput due to more computing resources being utilized for a single patient. Alternatively due to the views shared among the adjacent phases, it is possible to reuse the geometric coefficients calculations and to reconstruct multiple image volumes simultaneously. The proposed approach involves joint optimization of the MBIR cost-function to estimate images from all phases simultaneously. In this paper, the efficiency and feasibility of multi-phase reconstruction is investigated. First a quantitative metric is derived for estimating the improvement in the efficiency, then the improvement is verified using an implementation of the prototype algorithm. It is demonstrated using a cardiac multi-phase data that the proposed algorithm improves the computational efficiency with no change in the IQ compared to sequential MBIR reconstruction. It is also concluded that the improvement is dependant on the type of the optimization algorithm and the compute architecture.

Index Terms—Cardiac CT, multi-phase scan, MBIR, perfusion CT.

I. INTRODUCTION

Technology for X-ray detection in cone-beam (CB) geometry is rapidly improving and offers potential of performing fast high-resolution volume CT imaging. However, to optimally build such systems, the problem of CB image reconstruction needs to be fully understood. Statistical iterative methods (IR) exhibit particularly promise since it provides the flexibility of accurate physical noise modeling and geometric system description. Recently invented IR technique, model based iterative reconstruction (MBIR) [1], [2], significantly improves image quality (IQ) compared to conventional filtered back-projection (FBP). Iterative reconstruction suffers from an inherent challenge in terms of compute time compared to FBP due to multiple iterations needed for convergence and each iteration involves forward/back-projections using a complex geometric system model.

Multi-phase reconstruction using MBIR poses an additional challenge of reconstructing multiple images volumes corresponding to different phases. Multi-phase scanning involves acquiring CT data at different time instants and is widely used in a variety of applications such as cardiac and perfusion CT. In cardiac CT [3], [4], multi-phase scans are typically used to identify images at a quiescent phase with least motion artifacts. Reconstructions from multi-phase scan data can also be used to generate the motion signature which can then be applied to correct the motion artifact and in turn to improve temporal resolution [5]. A typical multi-phase scan in cardiac axial CT is demonstrated in Fig. 1(a), where view ranges for the adjacent phases partially overlap. In perfusion CT [6], multi-phase scan enables the potential to view crucial information such as blood flow (arterial and venous) and function in the heart, brain, joints and other parts of the body. In a non-cardiac axial perfusion CT geometry adjacent phases share the same view range and the data for all phases are totally co-located.

One simple way of reconstructing multiple volumes corresponding to different phases using MBIR is to reconstruct each phase sequentially, which is easy to implement but increases the reconstruction time by a factor of \( k \), which is the number of phase. If several parallel computing nodes are available, reconstruction for each phase can be distributed amongst different nodes and all phase reconstructions are performed simultaneously. In this case, a \( K \)-time speed up is gained compared to the sequential reconstructions. However, utilizing more compute resources for a single exam reduces the patient throughput which is critical in the CT work flow. A more efficient approach is proposed in this paper in which the image volumes corresponding to different phases are jointly reconstructed on the same compute node as described in section II-A. The efficiency is gained due to the fact that the views among adjacent phases are shared and hence the geometric model calculations can be shared during the forward/back-projections. Additional details about the memory organization of the sinograms are shown in section II-B. The improvement in the efficiency is demonstrated using cardiac multi-phase data in section III.

II. METHODS

A. Multi-phase reconstruction framework

The joint objective function for a multi-phase scan can be constructed as:

\[
\Phi(\mu) = \sum_{k=1}^{K} \sum_{i=1}^{M} (y_{ik} - \sum_{j=1}^{N} a_{ikj} \mu_{kj})^2 w_{ki}
\]

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of one-dimensional functions. In the case of multi-phase reconstruction, the $K$-dimensional vector $\mu_k$ is estimated by solving the function,

$$
\mu_k^{n+1} = \arg \min_{\mu_k \geq 0} \left\{ \theta_1,k_j(\mu_{k,j} - \mu_k^n) + \frac{1}{2} \theta_2,k_j(\mu_{k,j} - \mu_k^n)^2 + \beta \sum_{m \in N_j} \kappa_{k,j} \kappa_{k,m} \rho(\mu_{k,j} - \mu_k^m) \right\},
$$

(2)

which leads to the update step,

$$
\mu_k^{n+1} = \mu_k^n - \frac{\theta_1,k_j + \sum_{m \in N_j} \kappa_{k,j} \kappa_{k,m} \rho(\mu_{k,j} - \mu_k^n)}{\theta_2,k_j + \sum_{m \in N_j} \kappa_{k,j} \kappa_{k,m} \rho(\mu_{k,j} - \mu_k^n)}.
$$

(3)

where the first and second derivative of $\rho$ is computed using the functional substitution method [2]. $\theta_1,j$ is a $K$-dimensional gradient vector and $\theta_2,j$ is a $K$-dimensional diagonal matrix for voxel index $j$. The elements of these matrices are computed as,

$$
\theta_1,k_j = \sum_{i} a_{i,j} e_{k,i},
$$

$$
\theta_2,k_j = \sum_{i} a_{i,j}^2 e_{k,i}.
$$

(4)

e_k$ is the error sinogram and is maintained separately for every phase. The error sinogram is first computed as,

$$
e_{k,i} = \sum_{j} a_{i,j} \rho_k^{0} - y_i,
$$

(5)

and it is updated after every voxel update,

$$
e_{k,i} = e_{k,i} + \Delta e_{k,i},
$$

$$
\Delta e_{k,i} = a_{i,j} (\mu_k^{n+1} - \mu_k^n).
$$

(6)

The efficiency in the multi-phase reconstruction can be achieved by reusing the system coefficients $a_{i,j}$ to compute the error sinogram for all phases. The error sinogram is then consumed to compute the first and second derivatives of the cost-function as shown in eq. 4. However, this approach requires more memory than a sequential processing or parallel processing. The proposed method requires $K$ $M$-dimensional error sinogram, and statistical weight and $K$ $N$-dimensional image volume. The efficiency improvement in a multi-phase framework depends on the number of coefficients shared amongst the phases. For example, in a clinical scan with an average heart rate of 65 bpm and gantry rotation speed of 0.35 s over 984 views, a three phase scan with 5% phase difference will have total 1080 views. A very crude calculation of the efficiency ratio using the proposed approach is $\eta = \frac{422\times 3}{1080} = 1.78$. In the subsequent sections, a more accurate metric is used to compute the efficiency achieved with the proposed approach.

B. Memory organization and voxel processing

The update equation in eq. 3 consists of multiple steps of data operation and memory access in a general computer model shown in Fig. 3 where the local buffer could be on-chip SRAM or cache, Mem represents main memory.
During each voxel selection is used to speed up convergence by focusing the shared system matrix coefficients.

A burst mode to update the element which brings in all other elements automatically by reading weight as a package and to store it in a continuous physical memory line. By using the burst mode, reading weight can take as high as 80% of total computing cost. Therefore, it is crucial to organize the data in a more efficient way. A potential memory arrangement is to interlace the phase data to fill in one cache line, i.e., fetching one system matrix inside a zROI of dimension 13.

A cardiac CT clinical data is used to demonstrate the feasibility of multi-phase MBIR image reconstruction. The data is acquired in GE Discovery 750HD using a pitch of 0.2 with a gantry rotation speed of 0.35 s. Images are reconstructed at 66%, 75% and 83% of the RR cycle. Individual MBIR reconstructions of the phases are also performed to generate reference results. This comparison is performed to perform a sanity check on the implementation. It is clear from Eq. 1 that the cost function of a single MBIR reconstruction should converge to the same image as the multi-phase reconstruction for a given phase. All the images are reconstructed in a 70 cm field-of-view with a 418 × 418 matrix inside a zROI of dimension 13. A total of 99 slices are reconstructed as MBIR requires extra slices to be reconstructed outside the zROI. 4 iterations of ICD are used to reconstruct all the images. The dimension of the projection data used for multi-phase and single MBIR reconstruction is 1088 × 888 × 64 and 642 × 888 × 64, respectively. As explained in section II-A, the size of the error sinogram and statistical weight sinogram for multi-phase reconstruction is 3 × 1080 × 888 × 64. In contrast, the size of these sinograms are the same as the projection data for a single phase MBIR reconstruction.

Fig. 5 and Fig. 6 show the IQ comparison of the central slice of the zROI using the proposed multi-phase MBIR and separate MBIR reconstructions at the two phase locations. Hence a sanity check on the implementation is performed demonstrating that within the same number of iterations multi-phase MBIR reconstruction produces IQ comparable to separate MBIR reconstructions performed at the given phases.

In Section II, a rough estimation on the acceleration ratio is presented. A more thorough metric to evaluate the efficiency gained by doing multi-phase reconstruction over separate MBIR reconstructions is described here. In the best case scenario when all the phases share exactly the same number of views (i.e. axial perfusion CT), fetching one system matrix coefficient can be reused K (number of phases) times and this would hold for every fetch operation. However typically this would depend on the amount of overlap between the phases. The metric can be specific to every detector element and will indicate the efficiency of the fetch operation for every detector element. For the sake of simplicity, this metric is averaged across all detector elements and is shown in Eq. 8 below.

\[ \eta = \frac{\sum_{k=1}^{K} \sum_{j} \left( \sum_{i} I_{i} \in k I_{i}(a_{i,j} \neq 0) \right) \sum_{j} \sum_{i} I_{i}(a_{i,j} \neq 0) }{\sum_{k=1}^{K} \sum_{j} \sum_{i} I_{i}(a_{i,j} \neq 0) } \]

III. EXPERIMENTS AND RESULTS

A. Image quality validation

In the above operations, memory access operations 1/2/3/4/5 can take as high as 80% of total computing cost. Therefore, it is crucial to organize the data in a more efficient way. A potential memory arrangement is to interlace the phase data as a package and to store it in a continuous physical memory line. By using the burst mode, reading weight \( w_{x,i} \) at any phase can also fetch other phase \( w_{y,i} \) to fill in one cache line, which can hide the memory latency to read other phase data. Fig. 4 demonstrates the memory organization for accessing data in a three-phase reconstruction. This allows reading one element which brings in all other elements automatically by a burst mode to update the 3-dimensional voxel vector using the shared system matrix coefficients.

Interleaved non-homogeneous (NH) and homogeneous voxel selection is used to speed up convergence by focusing the computation where it is most needed [2]. During each non-homogenous update, a set of voxel indices are selected based on their update magnitudes. In the multi-phase MBIR framework, the same set of voxel indices is used to update images from all phases. This set of voxel indices is selected based on the sum of update magnitudes from all phases as given below:

\[ m(j) = \sum_{i=1}^{K} |\mu_{k,i}^{n+1} - \mu_{k,i}^{n}|. \]
where $I$ is an indicator function; $I_{(a_{i,j} \neq 0)}$ counts the number of non-zero geometric coefficients corresponding to the $j$-th voxel and $i$-th detector index. $I_{i \in k}$ counts the number of phases that share detector index $i$. The ratio $\eta$ is the efficiency gained due to shared system coefficients. A ratio of 3 demonstrates the best-case scenario for a 3-phase reconstruction and with 100% overlap in the views, while in the case of the scan used in section III-A the efficiency metric is evaluated as 2.3. This implies that the performance of a 3-phase reconstruction on the scan used in the previous section will be $3/2.3 \approx 1.3$ times a single phase MBIR reconstruction.

### B. Practical efficiency measurement

The multi-phase MBIR algorithm is implemented and is run on a multi-thread environment to evaluate the actual improvement in the efficiency. The performance of a 3-phase MBIR reconstruction is measured to be $3/1.4 = 2.1$ times a single phase MBIR reconstruction. The reason of slower performance than the predicted one is due to time spent in memory access. It is to be noted that all the efficiency metrics computed in the previous sections ignore the memory access time. Since the multi-phase MBIR reconstructions need bigger sinograms than a single phase MBIR reconstruction, the memory access time may have a significant impact in negating the efficiency gained due to sharing the system matrix coefficients. In contrast, the best-case performance is measured to be $3/2.1 = 1.5$ times a single phase MBIR reconstruction. The best-case situation is shown in Fig. I(b).

### IV. Conclusion

In this paper, a multi-phase MBIR algorithm is presented for reconstructing data from a multi-phase scan. The multi-phase scans have overlapping view ranges and as a result the geometric model calculations are shared in the proposed algorithm reducing the compute time. It is shown that the multi-phase MBIR algorithm is a better alternative than performing individual MBIR reconstructions sequentially without impacting the final image quality. It is shown that the images obtained with individual MBIR reconstructions are almost identical to the images obtained with the multi-phase MBIR reconstruction algorithm. The minor difference in convergence rate can be caused due to the fact the update mask used in the non-homogenous update is different in the two implementations.

The efficiency improvement is measured with a 3 phase reconstruction at 66%, 75%, and 83% using a clinical dataset on a particular implementation of the algorithm available in-house. The efficiency is gated by the memory access as the multi-phase algorithm requires larger arrays for the sinograms. To optimize the memory access, the data from all the phases is interlaced into a continuous physical memory line which can then be accessed in a burst mode to update all the image volumes simultaneously. Finally the measurements are based on ICD algorithm which is a sequential algorithm and may have a different memory access pattern than a simultaneous update algorithms such as PCG, OS-SPS. In addition the efficiency achieved is highly dependant on the compute platform itself. In the future, more study is required to evaluate the efficiency for
techniques that are based on simultaneous updates. The image model used in this study is still an indicator function and can be expanded to include more kinetic models as proposed in [8] for cardiac multi-phase scans.

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